LA PHYSIQUE REVISITÉE

# Resolution of the inconsistency of the initial laws of electromagnetism, implications and perspectives

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La physique revisitée, Numéro 1, pages 14-23
Publié le 03 09 2025
DOI : 10.5281/zenodo.17116208


**ABSTRACT:** The inconsistency in the initial laws of electromagnetism inspired Maxwell, but it was not resolved because it was circumvented by the use of constants when creating the SI.The objective of this theoretical study is to resolve the inconsistency and fill this gap in electromagnetism.

Cartesian analysis was used. After a historical summary that recalls how the inconsistency was circumvented without being resolved, we observe that the failure to consider the speed of light in the initial laws is the cause of its presence in the "measurements" deduced from the expressions of these laws. This "encystment" is the origin of the inconsistency between the CGS EMU and CGS ESU measurement systems. The incompatibility of these systems is then mathematically eliminated by identifying this inappropriate presence and removing it.

It follows that the speed of light affects the intensity of the phenomena. The speed of light acts as a multiplier parameter for the electrostatic force and as a dividing parameter for the magnetostatic force. It is shown that the permeability and permittivity of vacuum are compound constants: $\mu_0 = K_{MKS-A}/c$ and $1/\varepsilon_0 = K_{MKS-A}.c$ and that the value of the vacuum impedance $Z_0 = K_{MKS-A}$ depends only on the construction of the unit system.

The physical interpretation of these results and their implications are particularly interesting perspectives.

**keywords:** permeability, permittivity, speed of light, CGS system

**RÉSUMÉ :** L'incohérence des lois initiales de l'électromagnétisme a inspiré Maxwell, mais elle n'a jamais été résolue puisqu'elle a été contournée par l'utilisation de constantes lors de la création du SI. L'objectif de cette étude théorique est de résoudre l'incohérence pour combler cette lacune de l'électromagnétisme.

Une analyse cartésienne a été utilisée. Après un résumé historique qui rappelle comment l'incohérence a été contournée sans être résolue, il est montré que la non prise en compte de la vitesse de la lumière, dans les lois initiales, est la cause de sa présence dans les "mesures" déduites des expressions de ces lois. Cet "enkystement" est à l'origine de l'incohérence entre les anciens systèmes de mesure CGS UEM et CGS UES. On élimine alors mathématiquement l'incompatibilité de ces systèmes en identifiant cette présence inappropriée et en la supprimant.

Il résulte que la vitesse de la lumière agit dans l'intensité des phénomènes : La vitesse de la lumière intervient comme paramètre multiplicateur dans la force électrostatique et comme paramètre diviseur dans la force magnétostatique. Il est démontré que la perméabilité et la permittivité du vide sont des constantes composées : $\mu_0 = K_{MKS-A}/c$ et $1/\varepsilon_0 = K_{MKS-A}.c$ et que la valeur de l'impédance du vide $Z_0 = K_{MKS-A}$ dépend uniquement de la construction du système d'unités.

Ces résultats n'ont jamais été établis, leur interprétation physique et leurs implications, sont des perspectives particulièrement intéressantes.

Lire cet article en Français : https://la-physique-revisitee.science/methode-analytique/

**Mots clés :** perméabilité, permittivité, vitesse de la lumière, système CGS

# 1. INTRODUCTION

## 1.1. Historical treatment of inconsistency

The inconsistency between the initial expressions of Ampère's magnetostatic force and Coulomb's electrostatic force led physicists in the 19th and early 20th centuries to develop several electrical unit systems (ESU, EMU, Gaussian units, and so on). The international system was then established and circumvented this problem using constants [1],[2]. This inconsistency is related to the speed of light. Giovani Giorgi, the initiator of SI, used two constants to adapt the initial laws to the MKSA units. This harmonization made it possible to circumvent the problem. Finally, the rationalization proposed by Oliver Heaviside introduced a $4\pi$ factor and led to the current SI expressions where Giorgi's two constants were replaced by two new rationalized constants: the vacuum magnetic permeability $\mu_0$ and vacuum electrical permittivity $\varepsilon_0$. The complexity of the problem explains why this inconsistency was circumvented, rather than resolved. Before studying this subject in detail, we examine the current consequences of this unresolved and circumvented inconsistency.

## 1.2. Current consequences of the unresolved inconsistency

The vacuum magnetic permeability $\mu_0$ and the vacuum electrical permittivity $\varepsilon_0$ are SI constants constants related to the speed of light by the following relationship:

$$\mu_0.\varepsilon_0 = \frac{1}{c^2} \tag{1}$$

We are still unable to provide details of the distribution of the speed of light in each of the two constants because the initial inconsistency has never been resolved. We are used to this uncertainty, however, the lack of knowledge of this distribution must not be accepted as inevitable. Indeed, this lack of knowledge comes from a problem that has yet to be resolved, so we can already rigorously consider the possibilities of solutions.

Equation (1) implies exhaustive way:

$$\mu_0 = K.\,c^{n-2} \qquad \text{and} \qquad \varepsilon_0 = \frac{1}{K}.\,c^{-n} \qquad n \in \mathbb{Z}$$

$n \in \mathbb{Z}$ because speed is a physical constant with units. More likely we can have $n \in [0; 2]$

The vacuum electric permittivity and vacuum magnetic permeability are the SI constants of electrostatic and electromagnetic forces, respectively:

$$F_C = \frac{1}{\varepsilon_0}.\frac{Q.Q\prime}{4\pi r^2} \qquad \frac{F_A}{L} = \mu_0.\frac{I.\,I\prime}{2\pi r}$$

Therefore, the speed of light is necessarily involved in at least one of these forces; however, this is not explicitly expressed.

## 1.3. Reactivation of an old problem

The logical consequence of this unresolved inconsistency motivated this study. Indeed, we found that the physical role of the speed of light in these forces is still not understood because it is not even identified, even though it is invisibly considered through the permittivity and permeability of the vacuum. This question was known to physicists until the middle of the 20th century, since the electrical measurement systems resulting from these two forces were incompatible; electrical measurements were made using CGS mechanical units (centimeters, grams and seconds), and there was a light-speed relationship between the units of the two systems. In 1945, "Le Journal de Physique et le Radium" published an attempt to solve the problem using dimensional analysis [3], but based on an unjustified and erroneous assertion, this analysis was not followed up. This problem was not solved in the past; it gave rise to several systems of electrical units and was finally circumvented in the 20th century after the slow process of SI gestation, which lasted from 1901 to 1948.

This study reactivates the old problem, which is complex, unresolved, circumvented, and forgotten.

## 1.4. Scientific process

All data needed for the analysis have existed since the advent of SI. The small number, particularly the age of the bibliographic references, is due to the subject itself, as it is the reactivation of an old problem. The problem is treated theoretically, and its resolution requires a thorough analysis before obtaining a clear vision. The complexity of the solution requires decomposition into several parts of lesser difficulty (Cartesian method), which are listed here:

1. A historical summary. It begins with the initial observation of the inconsistency and recalls the decisions that lead to SI expressions using the vacuum permittivity and permeability. This summary shows that the inconsistency was not resolved but worked around.

2. Identification of a gap in the initial expressions of the Ampère and Coulomb forces concerning the speed of light. This gap leads to celerity in the "measurements" deduced from the expressions of these laws ("encystment"). The historical chronology is then resumed using an unknown in each of the forces to compensate for this gap, thus providing the form of the expressions expected at each stage.

3. Analysis of the two old measurement systems ESU and EMU: This allows to identify the unjustified "encystment" of the speed of light in the mechanical expressions of ohm in these two systems.

4. The deduction of the two unknowns by removing the "encystment" and, consequently, the deduction of the complete expressions of the forces and two electromagnetic constants.

5. Assessment of the validity of the conclusions and reflection on their implications and perspectives.

## 2. SIMPLIFIED HISTORICAL SUMMARY

Knowledge of this historical summary [1] [2] is essential for subsequent a analysis, which will use, point-by-point, the stages of this chronology.

### 2.1. The inconsistency between Coulomb's and Ampère's laws

In 1856, Wilhelm Weber and Rudolph Kohlrausch communicated about an observation: the ratio between the measurement of an electric charge using Coulomb's electrostatic force (ESU system) and its measurement using Ampère's electromagnetic force (EMU system) gave the speed of light:

Electrostatic force (formulation of the time):

$$F_C = \frac{Q.Q\prime}{r^2} \quad \text{whithout SI factor:} \quad K_C = \frac{1}{4\pi\varepsilon_0} \tag{2}$$

Charges $Q$ and $Q$ are separated by a distance $r$.

Electromagnetic force (formulation of the time):

$$\frac{F_A}{L} = 2.\frac{I.\,I\prime}{r} \quad \text{whithout SI factor:} \quad K_A = \frac{\mu_0}{4\pi} \tag{3}$$

$F_A/L$ is the force per unit length exerted on straight, parallel and infinite conductors separated by a distance $r$. $I$ and $I'$ are the currents flowing in the two conductors.

Our electrical units did not yet exist, charges or currents were expressed in millimeters, milligrams and seconds, based on mechanical measurements (measures qualified as absolute as opposed to relative measures [4]). However, the measurements were inconsistent between the electrostatic (ESU) and electromagnetic (EMU) systems. Indeed, with the Coulomb force, the electric charge had the following dimension:

$$[Q_{UES}] = \left[\sqrt{F_C}.r\right] = M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-1}$$

With Ampere's force, the current had the following dimension:

$$[I_{UEM}] = \left[\sqrt{\frac{F_A.r}{2.L}}.\right] = M^{\frac{1}{2}}.L^{\frac{1}{2}}.T^{-1}$$

Either for the charge:

$$[Q_{UEM}] = M^{\frac{1}{2}}.L^{\frac{1}{2}}$$

The ratio of the dimensions of the charges is a speed, the measurement systems were not consistent. But very interestingly, for the same charge, Weber and Kohlrausch obtained a ratio equal to the speed of light (measured seven years earlier).

$$\frac{Q_{ESU}}{Q_{EMU}} = c \tag{4}$$

This ratio and Faraday's discovery of the magneto-optical effect 11 years earlier allowed Maxwell to understand the relationship between electromagnetism and light. He synthesized electromagnetism and demonstrated the propagation of electromagnetic waves; however, the inconsistency between the initial expressions of the forces was not resolved.

### 2.2. The measuring systems CGS-EMU and CGS-ESU

Without an initial inconsistency, there would have been only one measurement system. Beginning in 1862, the British Association for the Advancement of Science (BAAS) resumed the use of these two measurement systems for electrical quantities, but this time in CGS units (centimeters, grams and seconds):

1. The CGS-EMU system was mainly used. It is based on electromagnetic force (EMU = Electromagnetic units).

2. The CGS-ESU system was rarely used. It is based on electrostatic force (ESU = Electrostatic units).

No physical meaning could be interpreted in the combinations of mechanical units of electrical quantities; they contained roots of mass and length and with their ratio in light speed, the two systems were always incompatible.

### 2.3. The practical unit system based on the CGS-EMU

A system of practical units (volt, ohm, etc.), close to the needs of engineers, was gradually put in place from 1874 by the BAAS [5]. The units of this practical system correspond to decimal multiples of the units of the CGS-EMU system because it was the most widely used system. Below, we give the two units initially defined and the first that results from them:

- $1\ \mathrm{V} \rightarrow\ 10^8\ \mathrm{g}^{\frac{1}{2}}\ \mathrm{cm}^{\frac{3}{2}}\ \mathrm{s}^{-2}$, which was close to the Daniell cell voltage ($\sim\ 1,1$ V) used as a reference in telegraphy and industry.

- $1\,\Omega \rightarrow\ 10^9\ \mathrm{cm\,s}^{-1}$, which was close to the resistance standard of the industrialist Werner Siemens ($\sim 0,9536\ \Omega$).

- $1\ \mathrm{A}\ \rightarrow 10^{-1}\ \mathrm{g}^{\frac{1}{2}}\ \mathrm{cm}^{\frac{1}{2}}\ \mathrm{s}^{-1}$, which was a consequence of previous choices.

Therefore, the electromagnetic force is the basis for the definition of electrical units; however the problem that leads to inconsistency with the ESU system has not been resolved.

### 2.4. The MKSA of Giorgi

In 1901, Giovanni Giorgi proposed using MKS units (meter, kilogram, second) as fundamental units of mechanics and proposed using a fundamental unit of electricity. Because the objective was to preserve the practical units defined in the EMU system, the fundamental unit of electricity had to remain based on the electromagnetic force. The ampere was chosen because the first electrical quantity to be derived from this force is the intensity. Its initial definition by the BAAS ($1\ \mathrm{A}^2\ \rightarrow 10^{-2}\ \mathrm{g\,cm\,s}^{-2} =\ 10^{-7}\ \mathrm{kg\ m\ \ s}^{-2}$) provides the reference value for electromagnetic force ($2\ 10^{-7}\ \mathrm{kg\ m\ \ s}^{-2}$ for $L = r = 1$ m). It was necessary to adapt the expression of this force to the unit using a constant $K_A$.

$$\frac{F_A}{L} = 2.K_A.\frac{I.\,I\prime}{r}$$

The constant $K_A$ had to have a value that corresponded to this force of 1 A in the two conductors and with these lengths, which gave $K_A = 10^{-7}$ kg m s$^{-2}$ A$^{-2}$.

It was also necessary to adapt the Coulomb force to the MKSA units using a constant $K_C$:

$$F_C = K_C.\frac{Q.Q\prime}{r^2}$$

The coulomb was also defined in the EMU system (by Ampère's force). Therefore, Giorgi proposed the use of the ratio (in light speed) to adapt the Coulomb force expression for this unit. Thus, the constants $K_C$ and $K_A$ had to respect the light-speed ratio (4) of the charges ESU and EMU (squared, considering the product of charges or currents):

$$\frac{K_C}{K_A} = \left(\frac{Q_{ESU}}{Q_{EMU}}\right)^2 = c^2 \qquad \Rightarrow \qquad K_C = K_A.c^2$$

The unresolved problem of the initial inconsistency was circumvented using these constants.

## 2.5. The rationalization

Before the new system was adopted, Heaviside proposed using two new constants, $\mu_0$ and $\varepsilon_0$, and a $4\pi$ factor instead of $K_A$ and $K_C$.

$$\mu_0 = 4\pi\ K_A \qquad \text{and} \qquad \frac{1}{\varepsilon_0} = 4\pi\ K_C$$

This "rationalization" allowed a simplification of the writing of Maxwell's equations where this factor $4\pi$ which was unnecessarily present (Heaviside called it "the excrescence"... [6]) , was eliminated [7] [8].

See appendix: The concepts of vacuum permeability and permittivity.

As a counterpart to its disappearance from Maxwell's equations, $4\pi$ factor appeared in certain electrical formulas (Coulomb's laws, Biot and Savart's laws, etc.), where it took on the physical meaning of the solid angle of all space [9]. This gives the current SI expressions for the Ampère's and Coulomb's forces:

$$\frac{F_A}{L} = 2.\frac{\mu_0}{4\pi}.\frac{I.\,I\prime}{r} \qquad K_A = \frac{\mu_0}{4\pi}$$

$$F_C = \frac{1}{4\pi\varepsilon_0}.\frac{Q.Q\prime}{r^2} \qquad K_C = \frac{1}{4\pi\varepsilon_0}$$

This rationalization did not change Giorgi's MKSA unit system but led to new expressions. Given the link between the replaced $K_C$ and $K_A$ factors, we obtain in accordance with d'Alembert's wave equation and Maxwell's equations rationalized by Heaviside as follows:

$$\frac{1}{4\pi\varepsilon_0} = \frac{\mu_0}{4\pi}.c^2 \qquad \Rightarrow \qquad \mu_0.\varepsilon_0 = \frac{1}{c^2}$$

The problem of initial inconsistency has since been overshadowed by adaptation to MKSA units, which circumvented it, and rationalization. This problem, which has become invisible, has not been resolved yet.

# 3. HIGHLIGHTING A GAP. COMPENSATING FOR THIS GAP BY USING UNKNOWNS

At the end of the 18th century and beginning of the 19th century, the expressions of the Coulomb and Ampère forces were established from observable quantities (forces, lengths, charges and intensity). Therefore, the speed of light, measured in 1849 by Hippolyte Fizeau, could not be included in this category at that time and was never later integrated as such.

## 3.1. Highlighting a gap

### 3.1.1. An inconsistency where speed of light appears

The EMU and ESU measurement systems are based on the initial forms of the electrostatic and electromagnetic forces:

$$F_C = \frac{Q.Q\prime}{r^2} \qquad \frac{F_A}{L} = \frac{I.\,I\prime}{r}$$

Unlike the SI expressions, where it is invisibly considered through the vacuum permeability and vacuum permittivity, the speed of light was not included in the initial expressions used to establish the two CGS systems. The speed of light appeared in the measurement report, creating an inconsistency whose mechanisms must be understood.

### 3.1.2. Consequence of not taking speed of light into account

We look at the consequences of not considering a necessary constant in an expression with the following example: Suppose we want to calculate the mass $m$ of an object in space, located at a distance $d$ from the center of the earth (of mass $M_T$), from the gravitational force $F$. We would use the expression of this force:

$$F = G.\frac{m.M_T}{d^2} \qquad \Rightarrow \qquad m = \frac{F.d^2}{G.M_T}$$

If the gravitational constant is not considered in the expression of the gravitational force, the result of $m$ will be $G$ times too large, with the value and units included. The gravitational constant will thus be "encysted" in the result of $m$ which will have neither the correct value nor correct dimension.

For the same reason, the speed of light was "encysted" in the results of the EMU and ESU systems, and the relationship between the two allows us to conclude that this gap exists. However, this single piece of information is insufficient to specify whether the problem arises in both expressions or in only one.

## 3.2. Compensation by using unknowns

We are looking for the forms of expressions that we would have had if the speed of light had been correctly integrated into these laws.

Méthod : We will consider that the two forces are potentially incomplete using two unknowns: $CF_A$ for Ampère's force and $CF_C$ for Coulomb's force, which allows us to study this question without prejudging the results.

We return at the historical timeline:

### 3.2.1. Initial expressions expected

If, in the 19th century, the expressions of force by Ampère and Coulomb correctly accounted for the physical phenomenon involving the speed of light, we would obtained had the following initial expressions.

**Initial expressions expected**

$$\frac{F_A}{L} = 2.CF_A \,.\frac{I.\,I\prime}{r} \tag{5}$$

$$F_C = CF_C \,.\frac{Q.Q\prime}{r^2} \tag{6}$$

Reminder: it is not prejudged whether the speed of light c occurs in $CF_A$, $CF_C$ or both.

At that time, currents and charges would still have been expressed in mechanical units from the expressions for forces. However, the speed of light would not have been ”enkysted” in the results.

**Consequence:** There would have been only one system of electrical measurements with mechanical units, which we call the ”unified system”.

A system of electrical measurements using mechanical units only exists by omission. This exists only if constant of adaptation to the respective units of the electrical and mechanical domains is not used. ”*It is in fact this constant which allows us to make the dimensional separation in relation to mechanical quantities*” wrote Ake Thulin in 1966 [10], (he was talking here about $\mu_0$ since it was a question of the rationalized SI and the practical units are defined in relation to the EMU system).Without such a constant, electrical quantities have mechanical dimensions that have no physical meaning. The ESU and EMU systems added a second omission: speed of light. An adaptation constant existed for each system ($K_A$ and $K_C$). Indeed, the speed of light does not intervene in the same way in the two systems since they are incoherent.
The unified system is the one which falls under the expressions 5 and 6:

- It is a system of electrical measurements with mechanical units because it has no constant adaptation to the units.
- It is a unified system because it takes into account the speed of light c.

### 3.2.2. Giorgi's MKSA expressions expected

Giorgi would have adapted the expressions of both forces to MKSA units without using the $c^2$ ratio. Therefore, he would have used a single dimensioning constant $K_{AC}$, which is common to both forces, instead of $K_A$ and $K_C$:

$$K_{Coulomb} = K_{Ampere}\;(K_{AC}) \;\text{ instead of }\; K_C = K_A.c^2$$

Giorgi's MKSA expressions would have taken the following forms:

$$\frac{F_A}{L} = K_{AC}\,.\,2.CF_A\,.\frac{I.\,I\prime}{r} \tag{7}$$

$$F_C = K_{AC}\,.CF_C\,.\frac{Q.Q\prime}{r^2} \tag{8}$$

### 3.2.3. Rationalized MKSA expressions expected

Rationalizing the system with the factor $4\pi$ would have given a single constant:

$$K_{MKS-A} = 4\pi\; K_{AC}$$

$$\text{instead of :}\qquad \mu_0 = 4\pi\;K_A \;\;\text{ and }\;\; \frac{1}{\varepsilon_0} = 4\pi\;K_C$$

We would have obtained the same rationalized SI system but with different formulations for the two forces:

$$\frac{F_A}{L} = K_{MKS-A}\,.\,CF_A\,.\frac{I.\,I\prime}{2\pi.r} \tag{9}$$

$$F_C = K_{MKS-A}\,.\,CF_C\,.\frac{Q.Q\prime}{4\pi.r^2} \tag{10}$$

The constant $K_{MKS-A}$ is a constant for rationalization and adaptation to MKSA units of complete expressions (rationalized and taking into account the speed of light).

### 3.2.4. Consequences : expected relations

Because it is the same rationalized system of units, the expressions are equivalent to those of the SI. The constant $K_{MKS-A}$ is currently invisible because it is integrated into the vacuum permeability and vacuum permittivity:

$$\mu_0 = K_{MKS-A}\,.\,CF_A \tag{11}$$

$$\frac{1}{\varepsilon_0} = K_{MKS-A}\,.\,CF_C \tag{12}$$

The following relationships can be deduced:

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = K_{MKS-A}\,.\,\sqrt{CF_A.CF_C} \tag{13}$$

$$\frac{1}{\mu_0.\varepsilon_0} = \frac{CF_C}{CF_A} = c^2 \tag{14}$$

These three constants cannot be determined directly; however, the two old EMU and ESU systems can be analyzed to determine whether the speed of light is ”encysted” in one or both. Once the ”encystment” of the speed of light is apparent, we can deduce the constants $CF_A$ and $CF_C$ which must make these systems converge towards the unified system (3.2.1).

## 4. ANALYSIS OF THE TWO ANCIENT MEASUREMENT SYSTEMS

Notation: The acronyms EMU and ESU refer to the CGS units in the scientific literature. To avoid ambiguities, we use the abbreviations MAG and STAT to designate the electromagnetic and electrostatic measuring systems with MKS mechanical units.

### 4.1. Dimensional table of MAG and STAT systems (EMU and ESU)

| Quantities | STAT | MAG |
|---|---|---|
| I | $M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-2}$ | $M^{\frac{1}{2}}.L^{\frac{1}{2}}.T^{-1}$ |
| U | $M^{\frac{1}{2}}.L^{\frac{1}{2}}.T^{-1}$ | $M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-2}$ |
| P | $M.L^{2}.T^{-3}$ | $M.L^{2}.T^{-3}$ |
| R | $L^{-1}.T$ | $L.T^{-1}$ |
| Q | $M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-1}$ | $M^{\frac{1}{2}}.L^{\frac{1}{2}}$ |

**Table 1 :** Dimensions of STAT and MAG systems [1]

**Observations:** Electric power is the only one to be coherent; for the others, we have:

- $[I_{STAT}] = [U_{MAG}]$
- $[U_{STAT}] = [I_{MAG}]$
- $[R_{STAT}] = 1/\,[R_{MAG}]$

We note that a speed appears in a contradictory way between the two systems:

$$\text{Electromagnétic system:}\quad [I_{MAG}] = \left[\frac{U_{MAG}}{\text{speed}}\right]$$

$$\text{Electrostatic systèm:}\quad [I_{STAT}] = [\text{speed}\ .\ U_{STAT}]$$

This reversal of speed is representative of the subject that interests us. The mechanical dimension of the resistance is directly related to the problem because it is a speed in the MAG system and the inverse of a speed in the STAT system. We must now have the values of the practical electrical units in both systems. We will establish, in MKS units, those that will be useful.

### 4.2. The units, ampere and ohm, in the MAG system

The initial definitions of ampere and ohm in the UEM CGS units allow them to be established in MAG MKS units (by passing through the square for the ampere).

| SI Units | EMU CGS (def) | MAG MKS values |
|---|---|---|
| 1 A | $10^{-1}$ g$^{\frac{1}{2}}$ cm$^{\frac{1}{2}}$ s$^{-1}$ | $\sqrt{10}\ 10^{-4}$ kg$^{\frac{1}{2}}$ m$^{\frac{1}{2}}$ s$^{-1}$ |
| 1 Ω | $10^{9}$ cm s$^{-1}$ | $10^{7}$ m s$^{-1}$ |

**Table 2:** Ampere and ohm, in the MAG system

### 4.3. The units, coulomb and ohm, in the STAT MKS system

The practical units do not come from a definition in the electrostatic system but can be established. We calculate the Coulomb force for 1 C and a distance of 1 m. Subsequently, we apply the force obtained to the initial expression.

$$F_C = \frac{1}{\varepsilon_0}.\frac{Q.Q\prime}{4\pi.r^2}$$

$$Q = Q\prime = 1\ \text{C},\quad r = 1\ \text{m},$$

$$\varepsilon_0 = 8,854187\ 10^{-12}\ \text{N m}^2\text{C}^{-2}$$

$$\Rightarrow\quad F_C\ \sim\ 8,987\ 10^9\ \text{N} = 8,987\ 10^9\ \text{kg m s}^{-2}$$

In STAT system (initial expression):

$$F_C = \frac{Q.Q\prime}{r^2}\quad\Rightarrow\quad 1\text{C}_{\text{STAT}} \sim\ 9,48\ 10^4\ \text{kg}^{\frac{1}{2}}\ \text{m}^{\frac{3}{2}}\ \text{s}^{-1}$$

We deduce the values of ampere (1 C.s$^{-1}$), volt (1 W.A$^{-1}$ with 1 W = 1 kg.m$^2$.s$^{-3}$), and ohm (1 V.A$^{-1}$), which give for ohm and coulomb:

| SI Units | STAT MKS values |
|---|---|
| 1 C | $9,48\ 10^4$ kg$^{\frac{1}{2}}$ m$^{\frac{3}{2}}$ s$^{-1}$ |
| 1 Ω | $0,111\ \ 10^{-9}$ m$^{-1}$ s $\sim\ \frac{1}{9}.10^{-9}$ m$^{-1}$ s |

**Table 3:** Coulomb and ohm, in the STAT system

### 4.4. Observation on the STAT and MAG values of the ohm

| SI Units | STAT MKS values | MAG MKS values |
|---|---|---|
| 1 Ω | $\frac{1}{9}.10^{-9}$ m$^{-1}$ s | $10^7$ m s$^{-1}$ |

**Table 4:** Ohm values in STAT and MAG systems.

This table and the approximation $c \sim 3.10^8$ m s$^{-1}$ allow us to write:

**Ohm in STAT and MAG systems**

$$1\ \Omega_{\text{STAT}} \sim \frac{1}{30\ c}\qquad \text{and}\qquad 1\ \Omega_{\text{MAG}} \sim \frac{c}{30}$$

The expressions STAT MKS and MAG MKS of ohm provide us with two pieces of information.

1. - We note the presence of a factor of 1/30 in both cases. This factor is the consequence of the choice of ohm by the BAAS in 1874 at $10^9$ CGS UEM units, that is, $10^9$ cm.s$^{-1}$.

2. We see, with the speed of light highlighted, that it is "encysted" in the STAT and MAG expressions of the ohm. This allows us to conclude that the speed of light should have be considered in the expressions of the two forces.

**Conclusion**

The speed of light must be considered:

- In the constant $CF_A$ of the Ampere force, to no longer be encysted in the expressions of the MAG units.
- In the constant $CF_C$ of the Coulomb force, to no longer be encysted in the expressions of the STAT units.

After these taken into account, the magnetic and electrostatic units will be identical (unified system).

# 5. CONSIDERATION OF THE SPEED OF LIGHT

## 5.1. Determination of constants $CF_A$, $CF_C$ and $K_{MKS-A}$

### 5.1.1. Highlighting the speed of light in the MAG expression of the ampere

We have already expression of ampere in the MAG system (Table 2); however, it is difficult to detach the speed of light, unlike in the case of ohm. Therefore, we use expression of ohm whose "encystment" of the speed of light is now apparent.

$$1\ \Omega = 1\ \text{W A}^{-2} = 1\ \text{kg m}^2\ \text{s}^{-3}\ \text{A}^{-2}$$

$$\Rightarrow\ 1\ \text{A}^2 = \frac{1\ \text{kg m}^2\ \text{s}^{-3}}{1\ \Omega}$$

$$1\ \Omega_{\text{MAG}} \sim \frac{c}{30}\ \Rightarrow\ 1\ \text{A}_{\text{MAG}}{}^2 \sim \frac{30}{c}\ \text{kg m}^2\ \text{s}^{-3} \qquad (15)$$

The encystment of the speed of light now appears for the square ampere.

### 5.1.2. Determination of constant $CF_A$

Without $CF_A$ (MAG system) we have:

$$\frac{F_A}{L} = 2.\frac{I.\,I\prime}{r} \quad \Rightarrow \quad \frac{r}{2L}.F_A = I.\,I\prime \qquad (16)$$

According to this last relation (16), the MAG expression of ampere (15) corresponds to the conditions of force and lengths such that

$$\frac{r}{2L}.F_A = 1\ \text{A}_{\text{MAG}}{}^2 \sim \frac{30}{c}\ \text{kg m}^2\ \text{s}^{-3} \qquad (17)$$

With $CF_A$ (Unified system, exp. (5)) we have:

$$\frac{F_A}{L} = 2.CF_A\,.\frac{I.\,I\prime}{r} \quad \Rightarrow \quad \frac{r}{2L}.F_A = CF_A\,.I.\,I\prime$$

Thus, in the unified system, for currents of 1 A and under the same conditions of force and lengths as in equation (17), we obtain

$$CF_A\ .\ 1\ \text{A}_{\text{uni}}{}^2 \sim \frac{30}{c}\ \text{kg m}^2\ \text{s}^{-3}$$

The value 30 was derived from the historical choice of ohm. The $CF_A$ constant must eliminate the "encystment" of speed of light in the ampere expression. We deduce:

$$CF_A = \frac{1}{c} \qquad (18)$$

We also deduce the expression of the ampere in the unified system:

$$1\ \text{A}_{\text{uni}}{}^2 \sim 30\ \text{kg m}^2\ \text{s}^{-3} \quad \Rightarrow \quad 1\ \text{A}_{\text{uni}} \sim \sqrt{30}\ \text{kg}^{\frac{1}{2}}\ \text{m s}^{-\frac{3}{2}}$$

It is recalled that the dimensions of an electrical measurement system with mechanical units have no physical significance. If, out of curiosity, we are still interested in the dimensions of the unified system, we find mechanical power roots for voltage and current and a dimension "1" for resistance (a simple proportionality factor between the two previous ones). Despite the substitution theorem [11], this equality of the dimensions of current and voltage would seem totally counterintuitive if we attributed a meaning to these dimensions. This probably explains why 19th century physicists did not solve this problem: rightly, the dimensions of current and voltage could not be the same, but they were looking for a mechanical expression of electricity, so they could not conceive of these results. Regarding this fruitless search, Arnold Sommerfeld stated in 1935: "*The absolute system (of the three mechanical units) could be considered unavoidable, as long as one could hope to deduce electricity from mechanics. This time is over.*" [2]

### 5.1.3. Deduction of the constant $CF_C$

The same reasoning can be repeated to determine the $CF_C$, but it is simpler to use (14), which leads to the same result:

$$\frac{CF_C}{CF_A} = c^2 \quad \Rightarrow \quad CF_C = c^2.CF_A$$

$$CF_C = c \qquad (19)$$

#### 5.1.4. Identification of $K_{MKS-A}$

We recall relation (13):

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = K_{MKS-A} \, . \, \sqrt{CF_A.CF_C}$$

From the constants $CF_A$ and $CF_C$ determined, we deduce:

$$K_{MKS-A} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sim 376,73 \text{ kg m}^2 \text{ s}^{-3} \text{ A}^{-2}$$

The constant of adaptation to the MKSA units and rationalization corresponds to the wave impedance in a vacuum.

### 5.2. Solutions of expected expressions

Equations (11) and (12) for the vacuum electric permittivity and vacuum magnetic permeability (3.2.4) give the following results.

$$\mu_0 = K_{MKS-A} \, . \, \frac{1}{c} \qquad \frac{1}{\varepsilon_0} = K_{MKS-A} \, . \, c \quad (20)$$

It now appears that the vacuum electric permittivity and vacuum magnetic permeability depend on the speed of light. These two constants are not magnetic and electric characteristics of vacuum; they express the manner in which the speed acts in each of these phenomena and consider the definition of the units. The terms permittivity and permeability only have meaning in media other than vacuum, for the relative values $\varepsilon_r$ and $\mu_r$.

The expressions of the two forces can be deduced:

$$\frac{F_A}{L} = K_{MKS-A} \, . \, \frac{1}{c} \, . \frac{I.\, I\prime}{2\pi.r} \quad (21)$$

$$F_C = K_{MKS-A} \, . \, c \, . \frac{Q.Q\prime}{4\pi.r^2} \quad (22)$$

The speed of light is a dividing parameter for the magnetic force and a multiplying parameter for the electrostatic force.

### 5.3. Independent expression of $K_{MKS-A}$

The vacuum electric permittivity and vacuum magnetic permeability are combinations of the speed of light and the constant $K_{MKS-A}$. Therefore, the latter can no longer be expressed from the previous ones and we must establish an independent expression. The initial definition of the ampere from an electromagnetic force of $2 \,.10^{-7}$N with $L = r = 1$m, allows us to express $K_{MKS-A}$:

$$2 \,.10^{-7} \text{ N} = K_{MKS-A} \, . \, \frac{1}{c} \, . \frac{1 \text{ A}^2}{2\pi}$$

Which gives the following result:

$$K_{MKS-A} = 4\pi \, . \, c \, . \, 10^{-7} \text{ N A}^{-2}$$

Because they were not considered in the initial expression of the electromagnetic force from which the EMU units were defined, factor $4\pi$, speed of light $c$, and coefficient $K_A = 10^{-7}$ N A$^{-2}$ constitute the constant that rationalizes and adapts the complete expressions to SI units.

$$K_{MKS-A} = 4\pi.\underline{c}.10^{-7} \text{ kg m}^2 \text{ s}^{-3} \text{ A}^{-2}$$

where $\underline{c}$ represents the MKS numerical value of the celerity of light in vacuum.

The constant $K_{MKS-A}$ appears in this analysis as a constant for adapting the expressions of forces to SI units, as Giorgi initially conceived the constants $K_A$ and $K_C$. It applies to rationalized expressions taking into account the speed of light which makes the MAG and STAT systems converge towards a single one. Therefore, this aspect is only that of adaptation to the SI of complete expressions. However, its physical meaning, independent of a system of units, is more complex. Indeed, this constant corresponds to the wave impedance in a vacuum.

## 6. CONCLUSIONS, EVALUATION AND PERSPECTIVES

Conclusions:

This analysis made it possible to identify how the speed of light must be considered in the expressions for the Ampère and Coulomb forces. The physical role of the speed of light in the fields intensities is no longer overshadowed by the vacuum permittivity and vacuum permeability. These constants do not express the electric and magnetic characteristics of the vacuum but correspond to the action of the speed of light in each of these phenomena. The speed of light not only imposes a delay in the propagation of potentials (Liénard-Wiechert) but also affects the intensity of the fields. From the expressions obtained for vacuum permittivity and vacuum permeability, the speed of light is a parameter that acts as a multiplier in the electric field and as a divider in the magnetic field. This is an unprecedented result because considering delayed times does not make the speed of light a quantity that plays a role in the intensity of the phenomena, such as, for example, the speed in a linear momentum.

Evaluation of the conclusions:

Regarding the vacuum permittivity and vacuum permeability, the previous conclusion is that they do not represent the characteristics of vacuum. This conclusion is consistent with the origin of these concepts, which is recalled in the appendix ("The concepts of vacuum permeability and permittivity"), where we can verify that they are only the artifices that compensate for the initial inconsistency and adapt the expressions to unit systems. This is the exact role that show their respective expressions (20).

Regarding the speed of light, which acts as a multiplier for the electric force and as a divider for the magnetic force, it is easy to verify that the intensity of these two forces corresponds to the influence of the speed of light. Indeed, for lengths systematically equal to one meter, Ampère's force is derisory for currents of 1 A (2 $10^{-7}$ N) and the Coulomb force is gigantic for charges of 1 C ($\sim$ 9 $10^9$ N).

perspectives:

The wave impedance in a vacuum expresses the ratio between the amplitudes $E$ and $H$ of the fields of an electromagnetic wave in vacuum; therefore, its role in the static laws of electromagnetism raises an interesting question to be explored further.

The physical actions that the speed of light represents in both cases are yet to be explained because this strictly analytical work provides no indication of this question. However, the expressions to which these actions lead are now defined, providing avenues for reflection on a new ways to understand the basic laws of electromagnetism. It is necessary to consider the propagation of action between charges in the formulation of the interaction forces. We can no longer approach them as simple remote actions materialized by the fields. We observed that the speed of light acts as a multiplier for the electric force and as a divider for the magnetic force. Therefore, the process that leads to the presence of an electric field and the process that leads to the presence of a magnetic field must be established considering this observation.

# ANNEXES

## A. The concepts of vacuum permeability and permittivity

### A.1. Magnetic permeability

In a medium with magnetic permeability $\mu$, the UNRATIONALIZED magnetic force is expressed as follows:

$$\frac{F_A}{L} = \mu.2\frac{I.\,I\prime}{r} \quad \text{with} \quad \mu = \mu_r.\mu_0$$

The magnetic permeability of vacuum $\mu_0$ serves as a reference for other media. It was initially equal to 1 in the CGS EMU system (see the initial expression (3) of $F_A$ in vacuum); therefore, permeability only had meaning in a medium but in a vacuum, its value 1 in the EMU system gave it the value $1/c^2$ in the CGS ESU system because of the inconsistency. Thus, the vacuum permeability acquired an existence because of the incoherence. Adaptation to MKSA units and rationalization (SI System) gave the vacuum permeability its current value.

$$\mu_0 = 4\pi\ 10^{-7}\ \text{kg m s}^{-2}\ \text{A}^{-2}$$

### A.2. Electrical permittivity

For electrostatics, Heaviside coined the term permittivity [8]. Permittivity corresponds to what Faraday called the "specific capacity for electrostatic induction" which Maxwell denoted as $K$ [12]. In a medium with dielectric permittivity $\varepsilon$, the UNRATIONALIZED electrostatic force is expressed as follows:

$$F_C = \frac{1}{\varepsilon}.\frac{Q.Q\prime}{r^2} \quad \text{with} \quad \varepsilon = \varepsilon_r.\varepsilon_0$$

The dielectric permittivity of vacuum $\varepsilon_0$ serves as a reference for other media. It was initially equal to 1 in the CGS ESU system (See initial expression (2) of $F_C$ in vacuum); therefore permittivity only had meaning in a medium but in a vacuum, its value 1 in the ESU system gave it the value $K = 1/c^2$ in the CGS EMU system because of the inconsistency. Thus, the vacuum permittivity acquired an existence because of the incoherence. The adaptation to MKSA units and rationalization (SI System) provides it its current name and value.

$$\varepsilon_0 = 8,854\ 187\ 82\ 10^{-12}\ \text{kg}^{-1}\ \text{m}^{-3}\ \text{s}^4\ \text{A}^2$$

### A.3. Synthesis

These results are summarized in the following table :

| systems | EMU | ESU | SI |
|---|---|---|---|
| $\mu_0$ | 1 | $1/c^2$ | $4\pi\ 10^{-7}$ kg m s$^{-2}$ A$^{-2}$ |
| $\varepsilon_0$ | $1/c^2$ | 1 | $8,854...\ 10^{-12}$ kg$^{-1}$ m$^{-3}$ s$^4$ A$^2$ |

**Table 1 :** Values of vacuum permeability and permittivity in UEM, UES and SI systems [9].

Unlike relative permeability and relative permittivity, which have physical meanings in media, the concepts of vacuum permeability and vacuum permittivity are only artifices that compensate for the initial inconsistency and adapt the expressions to unit systems.

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