LA PHYSIQUE REVISITÉE

The complicated history of electric units : implications on SI expressions

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This article on the history of science accompanies the scientific article:
Resolution of the inconsistency of the initial laws of electromagnetism, implications and perspectives

ABSTRACT: The International System of Units has only existed since 1946 and it was developed after a very long process that depended on the evolution of our understanding of electromagnetism.

This article aims to highlight the interconnections between mechanics and electromagnetism that have influenced the choice of units.

It is through a pedagogical approach to progress in understanding electrical phenomena that we will follow this construction of the system of units in general and of electrical units in particular.

it appears that the choice of electrical units was conditioned by the predominant use of the electromechanical measurement system based on magnetic force, CGS EMU, to the detriment of the measurement system based on electrostatic force, CGS ESU. These two systems were incompatible and had a ratio in the speed of light. Furthermore, the expressions for these forces failed to account for the solid angle 4π, which consequently appeared unjustifiably in Maxwell's equations; the rationalization proposed by Heaviside resolved this problem.

The article shows that this rationalization and incompatibility between these two systems necessitated, during their unification, the use of two constants to adjust the expressions for Ampère and Coulomb forces to the electrical units defined in the CGS UEM system. We call these constants the permeability and permittivity of free space.

keywords: EMU, ESU, International System of Units , CGS system

RÉSUMÉ : Le système international d'unités n'existe que depuis 1946 et il s'est élaboré après un très long processus qui a dépendu de l'évolution de notre compréhension de l'électromagnétisme.

Cet article vise à mettre en lumière les imbrications entre mécanique et électromagnétisme qui ont conditionnées les choix d'unités.

C'est par un cheminement pédagogique des progrès dans la compréhension des phénomènes électriques que l'on va suivre cette construction du système d'unités en général et des unités électriques en particulier.

Il apparait que les choix d'unités électriques ont été conditionnés par l'utilisation prépondérante du système de mesure électromécanique basé sur la force magnétique, le CGS UEM, au détriment du système de mesure basé sur la force électrostatique, le CGS UES. Ces deux systèmes étaient incompatibles et présentaient un rapport en vitesse lumière. D'autre part il manquait dans les expressions de ces forces la prise en compte de l'angle solide 4π qui en conséquence apparaissait de façon non justifiée dans les équations de Maxwell, la rationalisation voulue par Heaviside résolue ce problème.

L'article montre que cette rationalisation et cette incompatibilité entre ces deux systèmes a imposé, lors de leur unification, l'utilisation de deux constantes permettant d'ajuster les expressions des forces d'Ampère et Coulomb aux unités électriques qui avaient été définies dans le système CGS UEM. Nous appelons ces constantes perméabilité et permittivité du vide.

Mots clés : UEM , UES, système international d'unités, système CGS

1. FIRST DISCOVERIES

1.1. Year 1733

Charles DUFAY distinguished two different types of electricity: vitreous electricity (positive charge) or resinous electricity (negative charge) which repel each other when they are identical and attract each other when they are different.

Charles Dufay was the first to take a scientific approach to the electrical phenomenon, which until then had only served as a form of entertainment in salons. He published various memoirs in several fields (geometry, anatomy, chemistry, etc.).



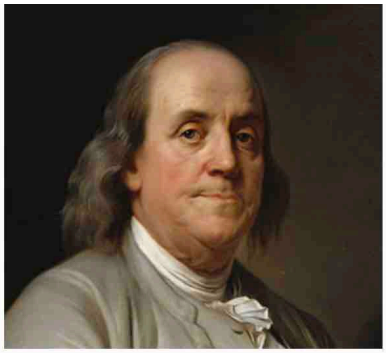
Charles DUFAY

Born in 1698
Died in 1739

1.2. Year 1750

Benjamin FRANKLIN developed the theory of the electric fluid in “plus” or “minus” (gain or loss).

Unfortunately, FRANKLIN lacks knowledge of DUFAY's work on the two types of static electricity. "Plus" and "Minus" are therefore not the new convention for designating them (a meaning they would later take on). These are terms that mean gain or loss of electricity. The notion of current is not far off.



Benjamin FRANKLIN in 1778

Born in 1706
Died in 1790

1.3. Year 1771

Henry CAVENDISH introduced the notion of “degree of electrification” (potential).

With Cavendish, the concepts of charges, potentials, and even resistances became clearer. He is therefore the precursor of Ohm's law.

Outside the electrical field, he was the first to measure the gravitational constant as well as the density of the Earth in 1798, inspired by the method used by COULOMB for electrical charges.



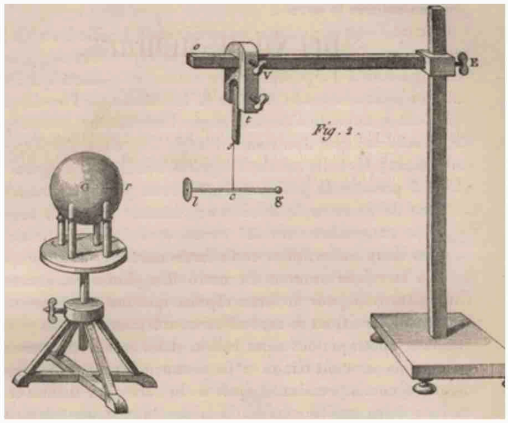
Henry CAVENDISH

Born in 1731
Died in 1810

1.4. Year 1785

Charles COULOMB established the law which gives the force of interaction in 1/d2 between two charges (first dimensional formulation of an electro-mechanical phenomenon):

”The repulsive force of two small electrified globes of the same type of electricity is inversely proportional to the square of the distance from the center of the two globes”





Charles COULOMB

Born in 1736
Died in 1806

Measurement of electrical force :

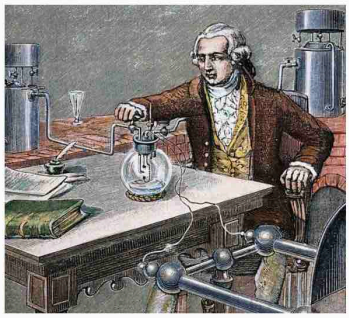
Coulomb used a trick to calculate the electrostatic interaction force: he measured the period of oscillations around the equilibrium position (the force being proportional to the square of the frequency).

2. FRENCH REVOLUTION AND REFORMS OF WEIGHTS AND MEASURES

The basis of mechanical units and future electrical units

2.1. Year 1789

Antoine LAVOISIER advocated for the decimal system and in 1790 a commission composed of BORDA, CONDORCET, LAGRANGE, LAVOISIER and TILLET, submitted the report that imposed the decimal system for currency. He had decimal mass boxes made for the precision balances he used in his chemistry.



The chemist LAVOISIER determining the formation of water.



Antoine LAVOISIER

Born in 1743
Guillotined in 1794

After the execution of LAVOISIER, under "the terror", the mathematician Louis LAGRANGE commented: "It only took them a moment to bring down this head and a hundred years, perhaps, will not be enough to reproduce a similar one."

2.2. Year 1791

The meter was defined by a commission composed of Borda, Condorcet, Lagrange, Laplace and Monge, as being 10 millionth of a quarter of the terrestrial meridian (meridian in the astronomical sense of the time: circumference). The reason for this choice was motivated by the fact that it was a basis independent of local susceptibilities, because it was common to all humanity (universalism). The measurement was carried out by triangulation..

The mathematicians and astronomers Pierre MECHAIN (1744-1804) and Jean-Baptiste DELAMBRE (1749-1822) measured the meridian by triangulation.







2.3. Year 1795

the decimal metric system was established by law. The kilogram is defined as the mass of one cubic decimeter of distilled water at 0°C. Standards are reproducible thanks to this definition.



Kilogram standard serving as an international reference kept at the International Bureau of Weights and Measures in Sèvres (under several Russian doll bells).

The division of time into hours, minutes and seconds, although not decimal, is not questioned because, unlike the units of length and weight, its use is already universal.

3. THE GREAT DISCOVERIES

3.1. Year 1800

Alessandro VOLTA invented the battery (electrolysis) and thus ended a controversy (one of the first ”scientific wars”) between the ”metallic electricity” he wanted to demonstrate and the ”animal electricity” of Luigi Galvani. The myth of ”animal electricity” would persist for a while and fuel science fiction with Mary Shelley’s 1818 novel, ”Frankenstein or the Modern Phenomenon.”



Voltaic pile

Stack of alternating zinc and silver discs, separated by cardboard or blotting paper soaked in salt water.

VOLTA designed this battery to demonstrate that electricity was not of biological origin but of chemical origin.



Alessandro VOLTA

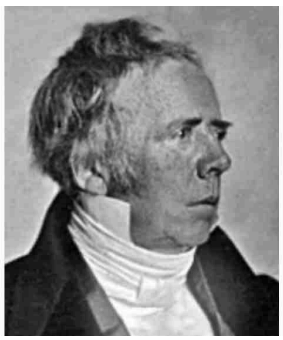
Born in 1745
Died 1827

3.2. April 1820

Hans Christian ØERSTED observed, during a lecture to his students, that the direction of a compass was deflected in the presence of an electric current. He did not yet have an explanation, but he had the good idea of publishing this observation immediately.

ØERSTED's observation triggered considerable enthusiasm in Europe and all its scientists looked into the question.

In reality, the magnetic effects of electricity were discovered in 1802 by Gian Domenico ROMAGNOSI, who published them, but unfortunately they were almost ignored.



Hans Christian ØERSTED

Born in 1777
Died in 1851

3.3. September 1820

André-Marie AMPÈRE became interested in the phenomenon observed by ØERSTED and discovered that the direction in which the compass needle moved depended on the direction of the electric current flowing nearby and deduced the so-called ”Ampère’s man” rule. He demonstrated the interactions between currents and attributed magnetism to the existence of electric currents, including those inside magnets; the phenomena were thus described as ”electrodynamic”.

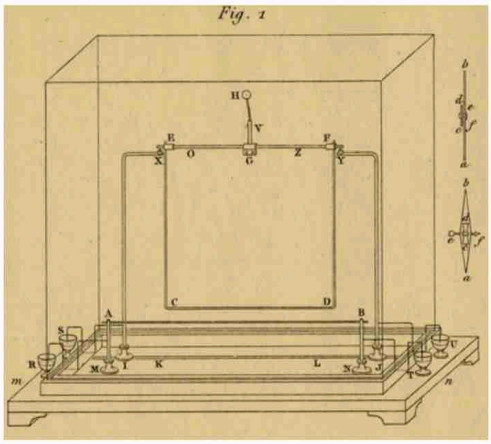


Figure of the device highlighting the electrodynamic force (interaction between currents).



André-Marie AMPÈRE

Born in 1775
Died in 1836

3.4. Year 1821

Michael FARADAY demonstrated ”an electromagnetic rotation” based on the interaction between a moving current conductor and a fixed magnetic field (the inverse of ØESTERD’s experiment), which was in fact the first electric motor.

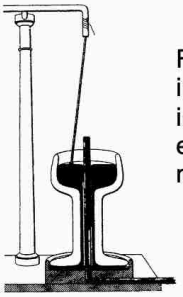
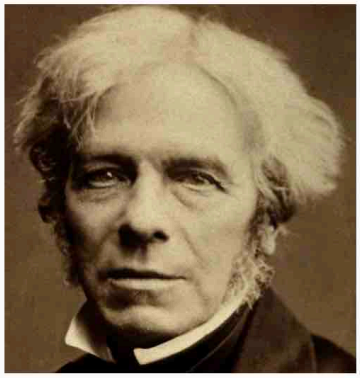


Figure of the "first" electric motor: A metal rod is immersed in a bath of mercury (conductive metal) in the center of which is a fixed magnet. When an electric current passes through the rod, it starts to move around the pole of the magnet.



Michael FARADAY

Portrait in 1842



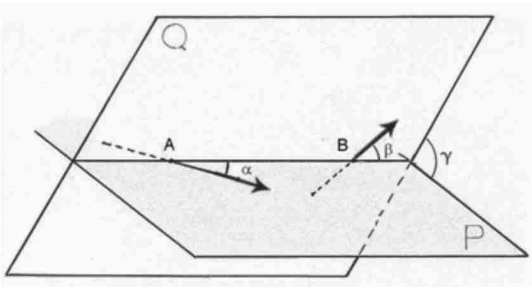
Michael FARADAY

Born in 1791
Died in 1867

3.5. Year 1826

AMPÈRE published his ”theory of electrodynamic phenomena” which were formalized mathematically for the first time: He expressed the forces of interaction between magnets and currents (Laplace Force) and the mutual forces between currents (AMPÈRE Force). He distinguished ”voltage electricity” (electric potential) from ”current electricity” (electrodynamics).

The formulation of the electrodynamic force (between currents) is a fundamental work based on four qualitative experimental facts using the “equilibrium case method” and allowing the development of the theoretical model1. It will lead to considerable progress.



AMPÈRE's fundamental formula expresses the force exerted on each other by two infinitesimal current elements I.ds and I'ds', placed at a distance r from each other and with relative orientations defined by the three angles α, β and γ.

The current element I.ds in A is located in the plane P.
The current element I'.ds' in B is located in the plane Q.

$$F = \frac{I\ I' ds\ ds' (\sin\alpha \sin\beta \cos\gamma - \frac{1}{2}\cos\alpha\cos\beta)}{r^2}$$

Reduced to the case where the planes P and Q are merged ($\gamma = 0$) and where the angles α and β are right angles, the force is expressed:

$$F = \frac{I\ I' ds\ ds'}{r^2}$$

ATTENTION :

The elements ds and ds' are infinitesimal, it is not the expression of a force with macroscopic lengths.

Details about AMPÈRE's force and the *"electrodynamic"* or *"electromagnetic"* formulations:

The expression of the *"electrodynamic"* force defined by Ampère concerns infinitesimal current elements ds and ds'. In this *"electrodynamic"* expression, ds and ds' should not be replaced by macroscopic length values (this would give a result that is half as small).

Progress in vector mathematical writing, as well as the joint use of LAPLACE's law and that of BIOT and SAVART, will allow a theoretical electromagnetic formulation of the forces acting on conductors.

Thus the *"electromagnetic"* force, also called AMPERE force, is a force per unit length, which for two parallel, rectilinear and infinite conductors, of direct currents I and I', distant from r, is expressed:

$$\frac{F}{L} = 2.\frac{I.\ I\prime}{r}$$ Without the factor $K_A = \frac{\mu_0}{4\pi}$ of the current SI system

3.6. Year 1827

Georg OHM established the law that bears his name using the conductivity of a conductor, its length and its cross-section and using the terms "current strength" for intensity, "difference of forces" for voltage and "conducting power" for conductivity.

$$U = \frac{1}{\sigma}.\frac{L}{S}.I$$



Geog OHM

Born in 1789
Died in 1854

3.7. Year 1831

Faraday discovered magnetic induction ("converting magnetism into electricity") and introduced the concept of a field. A year before him, in 1830, Joseph Henry had discovered the phenomenon of self-induction but had not published his results.

$$e = -N.\frac{d\phi}{dt}$$

LENZ-FARADAY Law (stated by Emil LENZ based on FARADAY's work): The electromotive force (induced voltage) is proportional to the number of turns N and the rate of change of the magnetic flux in the circuit. The - sign indicates that the induced emf opposes the cause that produced it :

4. THE ERA OF ELECTROMAGNETIC MEASUREMENTS

4.1. Year 1832

Carl Friedrich GAUSS proposed defining electrical measurements in millimeters, grams and seconds, since electromagnetic quantities are expressed by their mechanical interactions.

The basis of this measuring system is the AMPÈRE force which is expressed for parallel, infinite currents separated by r:

$$\frac{F}{L} = 2.\frac{I.I\prime}{r}$$

We see that in this measurement system, the intensity has the dimension of the square root of a force.

Gauss advocated for the decimal metric system, which he considered the most consistent for measurements in physical sciences.

Measurements made directly in relation to the basic units of mechanics are called absolute measurements because they give a direct measurement and not a measurement relative to another.

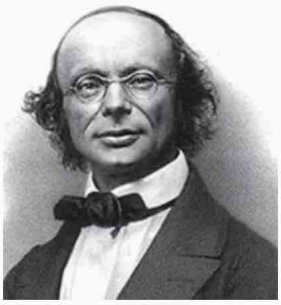


Carl Friedrich GAUSS

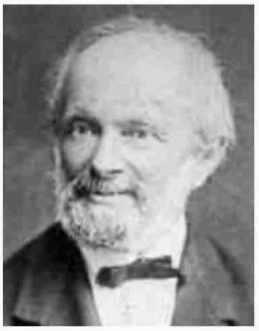
Born in 1777
Died in 1855

4.2. From 1837 to 1843

Gauss and Wilhelm Weber published the results of their numerous electromagnetic measurements carried out in the preceding years. The two physicists, friends in life despite their age difference, enthusiastically produced fundamental work for future measurement systems.



Wilhelm WEBER at the time of his work with GAUSS



Wilhelm WEBER

Né en 1804
Mort en 1891

4.3. Year 1843

Wilhelm Weber developed his electrodynamometer for accurately measuring Ampere's force. This device made it possible to more reliably link electromagnetic quantities to the units of mechanical quantities.

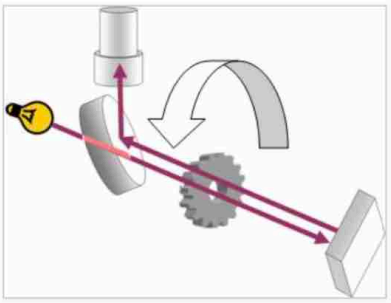


Electrodynamometer

The model opposite is the one built by the physicist Henri PELLAT in 1880. The force exerted between the outer coil and the inner coil is measured using a balance beam whose balance is sought using a sighting telescope (weighing).

4.4. Year 1849

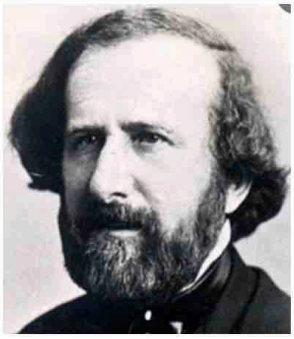
Hippolyte FIZEAU measured the speed of light at 315,000 km/s. The wave-like nature of light had been demonstrated by Thomas YOUNG in 1801.



Method of measuring the speed of light with a gear wheel.

The FIZEAU apparatus





At that time, there was still no well-established relationship between light and electromagnetism. However, four years earlier, in 1845, Michael Faraday had discovered the magneto-optical effect that bears his name (the Faraday effect).

Hippolyte FIZEAU

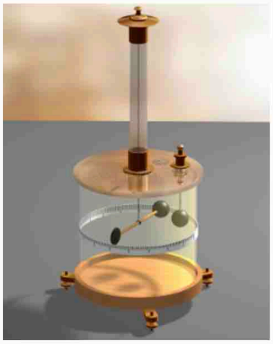
Born in 1819
Died in 1896

4.5. Year 1850

WEBER proposed using two measurement systems (still based on the millimeter, the gram and the second). The first for electromagnetism, already existing and now well calibrated, is that proposed by GAUSS and used since 1932 and a new one for electrostatics since mechanical interactions (electrostatic Coulomb's Law) give different relationships with mechanical quantities both in terms of values and units.

The basis of this electrostatic measuring system is Coulomb's Law which was expressed:

$$F = \frac{Q.Q\prime}{r^2} \quad \text{without the factor} \quad K_C = \frac{1}{4\pi\varepsilon_0} \quad \text{of the current SI system}$$



COULOMB torsion balance

4.6. Year 1856

WEBER noted with Rudolf KOHLRAUSCH that the values of the charges resulting from the electrostatic and electromagnetic measurement systems respectively are in a speed ratio and that this speed is that of light:

In the measurement system based on Coulomb's law, a charge is measured with a value Q_{es} and a dimension $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$ (M=Mass, L=Length, T=Time).

In the electromagnetic system, based on Ampere's force, an equivalent charge gives a different value Q_{em} with a dimension $M^{\frac{1}{2}}L^{\frac{1}{2}}$.

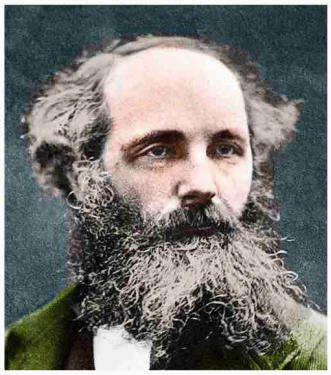
$$\text{We have in value and in units:} \quad \frac{Q_{es}}{Q_{em}} = \text{speed of light}$$

Therefore, there is an incompatibility between the two systems due to this dimensional inconsistency. Knowledge of this relationship allowed Giovanni Giorgi to propose the unification of the two measurement systems in 1901.

5. NEW THEORETICAL ADVANCES

5.1. Year 1865: Unification of electromagnetism by Maxwell

The discovery of the ratio in ”c” of the electric charges from the two systems, as well as the polarization of light by a magnetic field highlighted by FARADAY in 1845, inspired James Clerk MAXWELL:



James Clerk MAXWELL

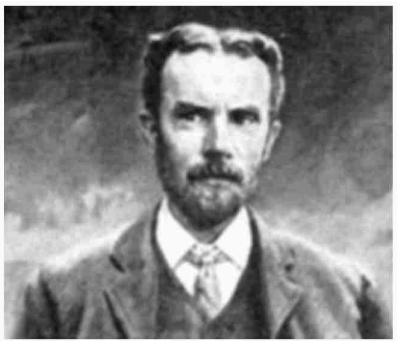
Born in 1831
Died in 1879

Based primarily on the work of AMPÈRE, whom he called the ”Newton of electricity,” but also of FARADAY and GAUSS, James Clerk MAXWELL developed a unified theory of electromagnetism that would have consequences, much later, in the middle of the 20th century, on the unit system. Light is identified as an electromagnetic wave.

In 1865, he presented a system of 20 equations which govern electromagnetism by introducing the notion of “displacement current” then in 1873 he reformulated his theory in the form of 8 equations.

5.2. Heaviside

In 1884, the self-taught and brilliant Oliver HEAVISIDE reduced MAXWELL’s 8 equations to 4, in a form corresponding to the needs of practical electrotechnical applications.



Oliver HEAVISIDE

Born in 1850
Died in 1925

Oliver HEAVISIDE also established the expression for the magnetic force applied to a moving charge (magnetic component of the Lorentz force).

He also suggested to Giovanni Giorgi, at the beginning of the 20th century, the use of the term dielectric permittivity ε_0 in expressions for the Ampere and Coulomb forces, in the definition of the future MKSA unit system.

But we’re not there yet.

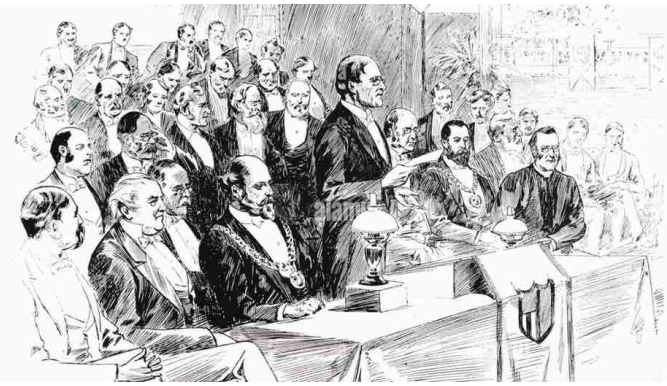
5.3. Summary of the electrical measurement situation at this stage

Until 1874, all electrical quantities were expressed as combinations of mechanical units (the millimeter, the gram, and the second) based on electromechanical interactions and were derived either from Ampère’s force (this is the Gauss system, the most widely used), or from Coulomb’s force (this is the system proposed by Weber). Electrical quantities did not yet have their own units. The ratio of identical charges between the two systems is the speed of light.

6. THE CGS MEASUREMENT SYSTEM AND UNITS FOR PRACTICE

6.1. Year 1874

Under the impetus of MAXWELL and THOMSON (Lord KELVIN), the CGS system (Centimeter, Gram Second) was proposed by the British Association for the Advancement of Science (BAAS) :



The British Association for the Advancement of Science, chaired by John Burdon-Sanderson in 1893.

In addition to adopting the decimal metric system for science and technology, the BAAS also introduced, in 1874, the prefixes from micro to mega to express decimal sub-multiples or multiples.

In this new system, mechanical and electrical quantities are expressed in terms of the units centimeter, gram and second. Inherited from the GAUSS and WEBER systems but in CGS, there are therefore two variants for electrical units: the CGS-EMU system (electromagnetic) which is the most used due to the historical implementation thanks to GAUSS and the CGS-ESU system (electrostatic) inherited from WEBER and which is in relation c or c^2, depending on the quantities, with the previous one:

Quantities	UES	UEM	Ratio $\frac{UES}{UEM}$
I	$M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-2}$	$M^{\frac{1}{2}}.L^{\frac{1}{2}}.T^{-1}$	c
U	$M^{\frac{1}{2}}.L^{\frac{1}{2}}.T^{-1}$	$M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-2}$	$\frac{1}{c}$
P	$M.L^2.T^{-3}$	$M.L^2.T^{-3}$	1
R	$L^{-1}.T$	$L.T^{-1}$	$\frac{1}{c^2}$
Q	$M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-1}$	$M^{\frac{1}{2}}.L^{\frac{1}{2}}$	c

Table : dimensions of EMU and ESU systems

The BAAS bases its system of units on Ampere's electromagnetic force (the CGS-EMU system) due to the historically predominant use of the electromagnetic measurement system initiated by GAUSS.

The BAAS formalizes Ohm's law: U=R.I

More practical units for industry are offered:

- The ohm is defined as 10^9 units of the CGS-EMU system, i.e. 10^9 cm/s. This new unit was more appropriate for the values used in industry, and was also close to within 5% of the resistance unit used in practice by the Germans, which they called Siemens, and which corresponded to the resistance of a mercury tube with a cross-section of 1mm^2 and 1 m long (1SE=0,9536 ohm).

- The volt (tribute to VOLTA) is defined as 10^8 units of the CGS-EMU system, that is to say $10^8\text{g}^{1/2}.\text{cm}^{3/2}.\text{s}^{-2}$ (Because this value is close to the voltage of the Daniell battery).

- The weber (which has nothing to do with the weber of the current SI) is the British ancestor of the ampere: it is linked to the volt and the ohm by Ohm's law: 1weber = 1 volt/1ohm.

The BAAS specifies the expression of continuous electrical energy: W=U.I.t

Indeed, whether in EMU or ESU units, we obtain the dimension of a work. Example: for one second, with the voltage and current values which today correspond to one volt and one ampere:

In CGS EMU:

$$10^8\ \text{g}^{1/2}.\text{cm}^{3/2}.\text{s}^{-2}\ .\ 10^{-1}\ \text{g}^{1/2}.\text{cm}^{1/2}.\text{s}^{-1}\ .\ 1s = 10^7\text{g}.\text{cm}^2.\text{s}^{-2}$$

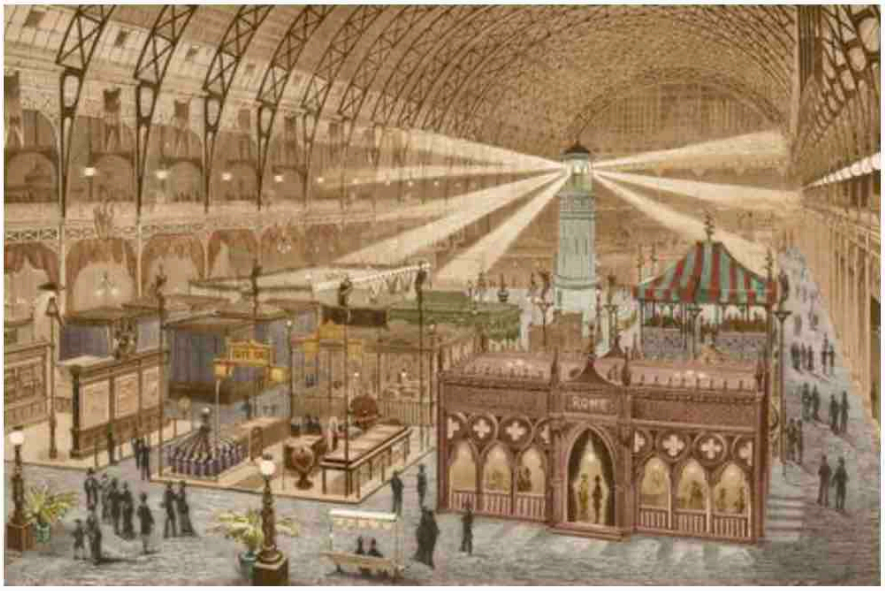
In CGS ESU:

$$1/3\ \text{g}^{1/2}.\text{cm}^{1/2}.\text{s}^{-1}\ .\ 3\ \text{g}^{1/2}.\text{cm}^{3/2}.\text{s}^{-2}\ .\ 1s = 10^7\text{g}.\text{cm}^2.\text{s}^{-2}$$

French physicists adopted this system because it was indirectly based on the decimal metric system and the BAAS decisions were sound for electrical units. The Germans retained the Gaussian system to which they were accustomed as well as their own units based on standards to which they were attached for practical reasons.

6.2. Year 1881

In 1881, the first international electricity exhibition took place in Paris.



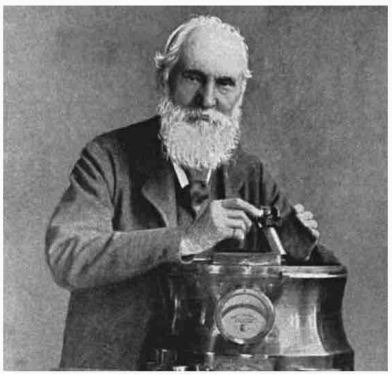
International Electricity Exhibition in Paris in 1881.

Palace of Industry

It was during this exhibition that the first International Congress of Electricity was held in the halls of the Trocadéro Palace.

Among the members of this congress was the illustrious physicist William THOMSON, better known as Lord KELVIN. He had, along with MAXWELL, driven the adoption of the CGS system by the BAAS.

In 1967, the kelvin became the unit of temperature in the SI system.



William THOMSON (Lord KELVIN)

Born in 1824
Died in 1907

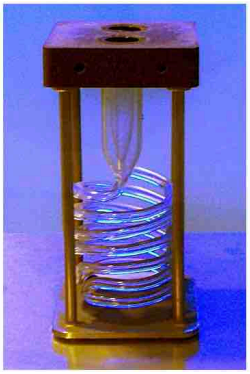
The Congress takes the following decisions:

- Adoption of the CGS system.
- The ohm and the volt retain their CGS-EMU values defined in 1874 by the BAAS (10^9 for the ohm and 10^8 for the volt).
- The standard ohm will be represented on the model of the German Siemens by a column of mercury of one square millimeter whose length must be adjusted by an international commission to correspond to its theoretical value (It will be 106 cm in 1884, refined to 106.3 cm in 1893).

Opposite, the German resistance standard (Siemens from 1860) which serves as a model for the ohm standard.

The unit siemens, abandoned as resistance, took on an inverse meaning in 1971 by becoming the SI unit of conductance:

$1S=1\Omega^{-1}$



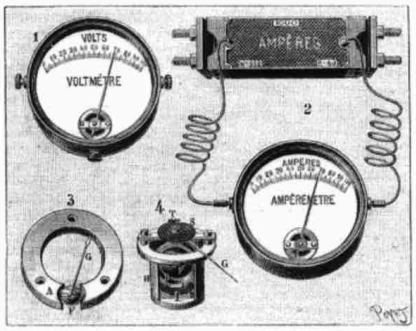
New electrical units are created, deduced from the ohm and the volt:

- The ampere, which is being used for the first time as a unit, will be the unit of current produced by one volt on one ohm. This definition therefore replaces the British weber because there already exists a German weber of similar but different value and there is a risk of confusion.

Amperemeter

The ampere is theoretically defined as 0.1 CGS-EMU unit or 0,1 $g^{1/2}cm^{1/2}/s$, but this measurement is difficult to achieve using Ampere's force, which is why it seems simpler to define it using the ohm and the volt.

The name weber as a unit of intensity was finally completely abandoned by the Germans and then taken up in 1946 in the MKSA system as a unit of magnetic flux.



- The Coulomb is the quantity of electricity delivered by a current of one ampere during one second.
- - The farad (tribute to FARADAY for his contribution to the study of dielectrics) is the capacity of a capacitor containing a charge of one coulomb for a potential of one volt at its terminals.

6.3. Year 1889

In 1889, at a new International Congress of Electricity in Paris, it was decided:

1. - The joule is the unit of work corresponding to a volt-coulomb (i.e. a volt-ampere-second). Indeed, according to the work-heat equivalence highlighted by James Prescott JOULE and validated by the BAAS in 1874: W= U.I.t therefore W=U.Q. In 1840, James Prescott JOULE demonstrated the electrical effect that bears his name, then the equivalence between heat and mechanical work in 1843.



James Prescott JOULE

Born in 1818
Died in 1889

2. - The watt is the unit of power, defined as the quotient of work by the time to produce it. The watt corresponds, continuously, to one volt-ampere (P=W/t= U.I). This last unit is named after the engineer James WATT.

James Watt was a man from the previous century. He was more of a great engineer than a physicist, as he worked primarily on steam engines, the technology of which he significantly advanced.

However, we owe to him, as well as to the chemists Joseph BLACK and Henry CAVENDISH, the distinction between the physical notions of latent heat and sensible heat.



James WATT

Born in 1736
Died in 1819

The choice of the name of this new unit is a outstretched handé to the mechanics with a view to harmonization, because they are holding their own congress in parallel.

In their congress, the mechanicians aligned their definitions of force, power and work, often used in place of each other, with the electricians:

- Power will therefore be the quotient of work by the time to produce it.
- The work will represent the product of a force by the path traveled.
- Unfortunately, despite pressure from the electricians, the mechanics refused to change their outdated system of units and kept the kilogram-force with a value of 9,81 $kg.m/s^2$, the kilogrammeter (work) and the horsepower (power).
- Energy is still perceived as a general quantity and retains its old units depending on the context (kilogram meter, calorie, etc.). All these archaic units will persist in the teaching of mechanics until the 1960s, forcing students to make painful conversions.



The Steam King whispers to the Coal King about the Electric Baby: "How big will it grow?"

An 1881 cartoon from the British satirical weekly "PUNCH" illustrating the concern of mechanics over the rise of electricians.

6.4. Year 1893

In 1893, the World's Fair took place in Chicago.

Chicago World's Exposition in 1893
(27 million visitors)

The first "Ferris wheel"



The International Electricity Congress meets again and decides:

- The henry is the unit of self-inductance giving a current variation of one ampere per second when a voltage of one Volt is applied to its terminals.
- - Units will no longer be defined by their theoretical values but by standards.

This last choice is unfortunately a big mistake because the standards will differ from one country to another and we will have to return to theoretical definitions later.

6.5. A happy chance in the choice of volt and ohm

We have seen that these two choices were initially guided by technological considerations. However, they would turn out to be the result of a happy coincidence during the transition to the MKS system. Indeed, the ampere being defined in 1881 by one volt over one ohm, it is worth:

$$10^{-1}\mathrm{g}^{1/2}\mathrm{cm}^{1/2}\mathrm{s}^{-1}$$

Thus the joule defined in 1889 by a volt-coulomb, that is to say a volt-ampere-second, gives:

$$1\ \mathrm{J} = 10^{8}\ \mathrm{g}^{1/2}.\mathrm{cm}^{3/2}.\mathrm{s}^{-2}\ .\ 10^{-1}\ \mathrm{g}^{1/2}\mathrm{cm}^{1/2}\mathrm{s}^{-1}\ .\ 1\ \mathrm{s} = 10^{7}\ \mathrm{g.cm}^{2}.\mathrm{s}^{-2} = 1\ \mathrm{kg.m.s}^{-2}.1\ \mathrm{m}$$

This corresponds to the work of a force of one newton (called MKS unit of force until 1960) on one meter. This perfect correspondence will facilitate the adoption in 1946 of the MKSA system proposed by Giovanni Giorgi in 1901. Indeed, this equality, without power of ten, between the mechanical $\mathrm{kg.m}^{2}.\mathrm{s}^{-2}$ and the joule, initially an electrical unit of energy, encouraged mechanics to adopt the joule and therefore the watt to the detriment of their archaic units (kilogram force, horsepower, etc.). The joule, although initially defined by electricity, was even defined by mechanics with this definition. The joule and the watt (which corresponds to the former to the nearest time), are the only units common to both fields. Other combinations of voltage units u_V and resistance u_R could have resulted in equal power:

$$1u_V.1u_I = 1\ \mathrm{kg.m}^{2}.\mathrm{s}^{-3} = 1\ \mathrm{W}$$

Any combination giving the following CGS EMU units ratio could have been suitable:

$$\frac{(1u_V)^2}{(1u_R)} = 10^{7}\ \mathrm{g.cm}^{2}.\mathrm{s}^{-3}$$

With u_V and u_R possible units of voltage and resistance expressed as powers of 10 of the CGS EMU units

But even if we remain within orders of magnitude close to technological needs, many combinations miss out on this 10^7. It is therefore notable that it is due to a degree of luck that we do not have powers of ten between the mechanical unit ($\mathrm{kg.m}^{2}.\mathrm{s}^{-3}$) and the volt-ampere. Through the Daniell battery and the Siemens standard, technology influenced this choice, which proved useful for harmonization with mechanics.

7. GIOVANNI GIORGI, FATHER OF THE MKSA MEASURING SYSTEM

7.1. Criticisms of the CGS system

1. The cm and the gram are submultiples of the defined units.

2. Although all the electrical units of the CGS-EMU system originally came from the measurement of the Ampere force and therefore from the unit of intensity, this is no longer the case since, on the contrary, the ampere is now defined from the volt and the ohm, themselves obtained by calibration. On the other hand, the standards vary from one country to another.

3. We always have a second system for electrostatic measurements (the CGS-ESU system) and the two systems give different values and units, the measurements of the charges are in a speed ratio and this speed is that of light.

7.2. Year 1901

The physicist Giovanni GIORGI wanted to eliminate all these drawbacks:

1. Return to a direct theoretical definition of mechanical and electrical quantities.

2. Unify the two measurement systems (electromagnetic and electrostatic) by introducing constants taking into account the speed-to-light ratio of the charges.

It was to the Italian Electrotechnical Association that Giovanni GIORGI presented his new system in 1901.



Giovanni GIORGI

Born in 1871
Died in 1950

He therefore proposes a new system of units based on the meter instead of the cm and the kilogram instead of the gram (these two units being defined directly) as well as the second and he suggests using the ampere as the new fundamental unit of electricity, the latter being defined by its theoretical value derived from Ampere's force.

GIORGI does not want to abolish the electrical units already in use. However, the ampere has been defined as one volt over one ohm, or $0,1$ g$^{1/2}$.cm$^{1/2}$.s^{-1} in the CGS EMU system and thus corresponds to an Ampere force of $0,02$ g.cm.s^2 according to the initial expression without a constant of the latter, or 2 10^{-7}kg.m.s^{-2}.

So this will be its definition in this MKS new system:

"The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2x10^{-7} kg.m.s^{-2} ***per metre of length. "*** (The Newton unit does not yet exist).

Note: The definition of the ampere was modified in the SI in 2019.

It is therefore necessary to review the expression of AMPÈRE's force and adapt it to this unit with a constant, K_A :

$$\frac{F_A}{L} = 2.K_A.\frac{I.\,I\prime}{r} \quad \text{with} \quad K_A = 10^{-7}\ \text{kg.m.s}^{-2}.\text{A}^{-2}$$

On the other hand, to unify the two measurement systems, it is necessary to adapt COULOMB's electrostatic law to the unit that bears his name. But since the coulomb comes from the electromagnetic system, it is necessary to add a constant K_C linking the two systems:

$$F_C = K_C.\frac{Q.Q\prime}{r^2}$$

Determination of K_C :

By the product Q.Q', there are coulombs squared, as in the force of AMPERE since one ampere is equal to one coulomb per second. Thus the ratio of the constants must respect the ratio of the charges (of the EMU and ESU systems) squared, this ratio being equal to the speed of light c:

$$\frac{K_C}{K_A} = \left(\frac{Q_{UES}}{Q_{UEM}}\right)^2 = c^2 \quad \Rightarrow \quad K_C = K_A.c^2$$

The idea of this new system is beginning to spread.

7.3. Rationalization

Oliver HEAVISIDE, who reduced MAXWELL's equations to four, then suggested to Giovanni GIORGI to use, in the expressions of the AMPÈRE and COULOMB forces, the terms μ_0 and ε_0, which he called "magnetic permeability and dielectric permittivity of the vacuum" (he invented the second term) and to rationalize the system by using a 4π factor. That's to say :

$$K_C = \frac{1}{4\pi\varepsilon_0} \qquad K_A = \frac{\mu_0}{4\pi} \qquad \text{and so} \quad \frac{K_C}{K_A} = \frac{1}{\mu_0\varepsilon_0} = c^2$$

Thanks to which:

1) 1) The formulation of the COULOMB force expresses that the influence of a charge Q on a charge Q', distant from r, is distributed over the surface of the sphere of radius r:

$$\vec{F} = \frac{1}{\varepsilon_0}.\frac{Q}{4\pi r^2}.Q'.\frac{\vec{r}}{r}$$

2) 2) The same applies to the magnetic field induced by a charge Q moving at speed $\vec{v}$ with the BIOT and SAVART law:

$$\vec{B} = \mu_0.\frac{Q}{4\pi r^2}.\vec{v} \wedge \frac{\vec{r}}{r}$$

3) 3) The factor 4π which appeared at the time in MAXWELL's equations disappears and only appears in electrical formulas where it has the meaning of a solid angle.

As a result of these choices:

$$\mu_0 = 4\pi.10^{-7}\ \text{kg.m.A}^{-2}.\text{s}^{-2} \quad \text{et} \quad \varepsilon_0 = \frac{1}{\mu_0 c^2} = 8,854\ 187\ 82\ .\ 10^{-12}\text{F.m}^{-1}$$

At this stage the MKSA system has still not been adopted but the idea of this new rationalized system is slowly gaining ground.

Anecdote : Heaviside railed against the 4π factor that appeared in Maxwell's equations: he considered it "particularly obnoxious and misleading" and called it "the excrescence", adding: *« Privately I use units which get rid of them completely, and then, for publication, liberally season with 4π's to suit the taste of B.A.* unit-fed readers.»* (*British Association). Extraits de : Electrical papers. V.1, 1892 : Macmillan and Co.

8. THE MKSA SYSTEM THEN SI

8.1. The MKSA system

In 1946, almost half a century after its initial proposal, at the 9th General Conference on Weights and Measures, GIORGI's MKS system with its definitions of electrical units was finally adopted as the international system of units (resolution 2).

The ampere with its theoretical definition (GIORGI 1901) is therefore added as a fundamental unit to this new system which thus becomes the MKSA system.

The unit of force (future newton) defined at 1 kg.m.s^{-2}, remains a derived mechanical unit without a name, called the MKS unit of force. The units of power and energy derived from electricity, the watt and the joule, now apply to mechanics and the joule is even defined by it, it therefore receives a definition that is no longer electrical: it corresponds to the work of the unit of force on one meter.

$$W = F.L \qquad 1\ J = 1\ \text{Kg.m.s}^{-2}\ .\ 1\ \text{m}$$

A power of one watt always corresponds to the work of one joule produced in one second.

$$P = \frac{W}{t} \qquad 1\ \text{W} = \frac{1\ \text{J}}{1\ \text{s}}$$

The volt is defined as the direct voltage which under one ampere gives one watt.

$$U = \frac{P}{I} \qquad 1\ \text{V} = \frac{1\ \text{W}}{1\ \text{A}}$$

The weber is defined as the magnetic flux in the circuit of a single turn (n=1), which produces an electromotive force of one volt during a uniform decay of one second.

$$e = -n.\frac{d\phi}{dt} \qquad 1\ \text{V} = -1.\frac{-1\text{Wb}}{1\ \text{s}} \qquad \Rightarrow \qquad 1\ \text{Wb} = 1\ \text{V.s}$$

8.2. The SI system

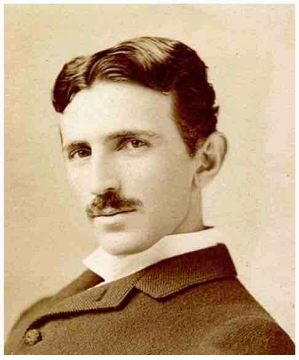
In 1954, the thermodynamic temperature unit (whose name kelvin was not actually given until 1967) and the candela were added as fundamental units.

In 1956, the MKSA system, after the addition of the kelvin and the candela as fundamental units in 1954, took the name ”International System of Units”.

In 1960, the MKSA unit of magnetic induction field (one weber/m2) was named tesla.

$$\phi = B.S \qquad 1\text{Wb} = 1\text{T}.\,1\text{m}^2 \qquad 1\text{T} = 1\,\frac{\text{Wb}}{\text{m}^2}$$

Nikola Tesla is an engineer and inventor who demonstrated the advantages of alternating current in the production, transmission, and distribution of electrical energy. We owe him our current system. He is also one of the inventors of the asynchronous motor, an electric motor suitable for over 80% of applications. He has approximately 300 patents to his name.



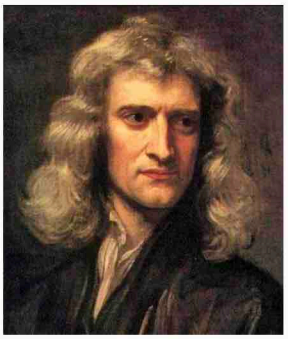
Nikola TESLA

Born in 1856
Died in 1943

At this same conference, the MKS unit of force finally takes that of newton: $1\text{ N} = 1\text{ kg.m.s}^{-2}$



Isaac Newton, became the character of a recurring comic strip gag under the pencil of the cartoonist Marcel Gotlib



Isaac NEWTON

Born in 1642
Died in 1726

Isaac Newton is among the last whose name is taken as a unit, yet he was one of the first great scientists and he inspired all the others.

In 1971, the mole was added as a fundamental unit.

In 2018, the seven base SI units received new, extremely precise definitions that moved them away from their original definition, with the ampere now defined by its actual meaning of electron flow.

Important note: In 2018, the ampere lost its definition based on Ampere’s force, but the electrical equations, adjusted by constants (permeability and permittivity) to the values expressed by this unit, remain. We must therefore never lose sight of the fact that the values of these constants are due to the choices made in the past to define this unit. If the choice of the unit of electric current had been different, we would still have:

$$\frac{1}{\mu_0 \varepsilon_0} = c^2$$

But it would be the values of $\mu_0$and ε_0 that would be different. This relationship is the consequence of the initial incompatibility between the EMU and ESU systems, noted by Weber and Kohlrausch in 1856 and the standardization of the laws during the transition to MKSA.

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