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# The plane wave impedance in vacuum $Z_0$ is an electromechanical coupling constant

Jean-Marc Augustin Roux[1]
[1]Professeur Agrégé

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**ABSTRACT:** Although the plane-wave impedance of vacuum $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is a fundamental constant of electromagnetism, it has historically been little used from a strictly theoretical point of view, most often appearing as a derived quantity. Its use is mostly limited to a role as a normalization or reference parameter in electromagnetic engineering applications. However, the recent resolution of the inconsistency in the initial laws of electromagnetism sheds new light on it.

This work makes it possible to deepen its interpretation and to show that its actual role is of considerable importance.

After clarifying the concept of wave impedance, possible misinterpretations are eliminated, and a study of its physical significance is carried out based on the role (effect or excitation) of electromagnetic fields.

It follows that the plane-wave impedance in vacuum is a constant related to the mechanical effects of electromagnetic phenomena.

This result, when confronted with the new expressions of the Ampère and Coulomb forces obtained through the resolution of the inconsistency in the original laws of electromagnetism, shows that it is more precisely an electromechanical coupling constant.

**keywords:** Vacuum impedance, permittivity, permeability, Coulomb force, Ampère force, Electromagnétic field

**RÉSUMÉ :** Bien que l'impédance d'onde plane du vide $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ soit une constante fondamentale de l'électromagnétisme, elle a historiquement été peu exploitée du point de vue strictement théorique, apparaissant en général comme une quantité dérivée. Son usage se limite le plus souvent à un rôle de paramètre de normalisation ou de référence dans les applications d'ingénierie électromagnétique. Cependant la résolution récente de l'incohérence des lois initiales de l'électromagnétisme lui donne un nouvel éclairage.

Ce travail permet d'approfondir son interprétation et montrer que son rôle réel est d'une importance considérable.

Après avoir précisé le concept d'impédance d'onde, les erreurs d'interprétation possibles sont éliminées et une étude concernant sa signification physique est réalisée à partir du rôle, d'effet ou d'excitation, des champs électromagnétiques.

Il résulte que l'impédance d'onde plane dans le vide est une constante liée aux effets mécaniques des phénomènes électromagnétiques.

Ce résultat, confronté aux nouvelles expressions des forces d'Ampère et Coulomb obtenues par la résolution de l'incohérence des lois initiales de l'électromagnétisme, montre qu'il s'agit plus précisément d'une constante de couplage électromécanique.

**Mots clés :** Impédance du vide, permittivité, permeabilité, Force de Coulomb, Force d'Ampere, Champ électromagnétique

# 1. INTRODUCTION

Physical constants generally occupy a central place in electromagnetic theory. It is therefore remarkable that the plane wave impedance in a vacuum, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$, remains largely underutilized, most often being reduced to the role of a simple reference for the characteristics impedances of material media. This marginalization contrasts with its fundamental nature, suggested by its definition based on electromagnetic constants. Such a situation indicates that its physical meaning is not fully understood. The present work aims to clarify this point by methodically discarding inadequate interpretations in order to better define the true nature of this constant. Recent developments have provided new insight that has facilitated this study, and it is the fundamental importance of this constant that will be brought to light. We will successively:

1. Perform an analysis based on its current definition and eliminate any misinterpretations that may arise.
2. Adopt a new approach by taking into account the role (effect or excitation) of electromagnetic fields, of which it is the ratio in a vacuum. This allows us to highlight its electromechanical nature.
3. Integrate recent developments that allow us to specify its function as an electromechanical coupling constant in interactions of electromagnetic origin.
4. Assess the validity of the conclusions and offer some reflections on the implications and perspectives.

# 2. ANALYSIS OF THE CONSTANT $Z_0$

## 2.1. Intrinsic impedance of a medium

### 2.1.1. The concept of impedance in its generalized meaning and use

The term impedance is used in its general sense to refer to the relationship between a stimulus and its response (mechanical impedance, acoustic impedance, etc.). The concept of impedance can be summarized as follows:

- Impedance is a ratio of coupled quantities.
- It applies to dynamic phenomena (sinusoidal, transient, or time-varying).
- It is complex to take into account the phase shifts.
- It can be generalized to different fields: electrical $(V/I)$, acoustic $(p/v)$, mechanical $(F/v)$, electromagnetic $(E/H)$, thermal $(\phi/T)$.

The concept makes it easy to determine the response of a system without using differential equations: at the interface of two propagation media, the impedances $Z_1$ and $Z_2$ of each of the media, enter into the calculation of the reflection coefficients $\mathcal{R}$ and transmission coefficients $1-\mathcal{R}$:

$$\mathcal{R} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \tag{1}$$

Examples:

- The role of the gel (acoustic impedance propagation medium $Z_1$) in an ultrasound scan is an impedance adaptation to decrease at the level of the skin (impedance propagation medium $Z_2$), the reflection of ultrasounds in favor of their transmission ($Z_1 \sim Z_2$) [1].
- At the end of a two-wire or coaxial electrical transmission line with a "characteristic" impedance $Z_1$, an electrical impedance of value $Z_2 \sim Z_1$ is placed to suppress reflection and thus prevent the creation of standing voltage and current waves in the line (risk of interference) [2]. This "characteristic impedance" is electrical:

$$Z_1 = \sqrt{\frac{R + jL\omega}{G + jC\omega}}$$

Where R, L, G, and C are respectively the linear resistance, inductance, conductance, and capacitance. For high frequencies (R and G negligible), we have $Z_1 \sim \sqrt{L/C}$, which is resistive. The impedance placed at the end of the line to suppress reflection is therefore a resistance.

Remarks :

1. A negative reflection coefficient corresponds to a reversal of sign of the reflected quantity.
2. For acoustics, the expression for the reflection coefficient (1) is given above for normal incidence (perpendicular to the surface separating the media), the same applies to electromagnetism.

### 2.1.2. Definition of intrinsic impedance

Intrinsic impedance is defined, in the absence of free charges, in a linear, homogeneous and isotropic medium, by the ratio of the electric field strength E to the magnetic excitation field H of a monochromatic progressive plane wave and is expressed as follows:

$$Z = \left(\frac{E}{H}\right)_{\text{MPPW}} = \sqrt{\frac{\mu}{\varepsilon}} \tag{2}$$

Where $\mu$ and $\varepsilon$ are respectively the dielectric permittivity and magnetic permeability of the medium. This relationship follows from Maxwell's equations. The two fields are perpendicular to each other and to the direction of propagation. Therefore, this relationship can be expressed more rigorously in vector form, taking into account the unit propagation direction $\vec{n}$:

$$\vec{E} = \sqrt{\frac{\mu}{\varepsilon}}\vec{H} \wedge \vec{n} \tag{3}$$

The intrinsic impedance is designated in English as characteristic impedance of a medium, but this term is discouraged in French by the IEC (IEC 705-03-23) [3]. This term is a source of confusion with the characteristic impedance of an electrical transmission line. The term "impedance" does not correspond in the present case to an electrical interpretation; it is used in the general sense of the term [4].

## 2.2. Plane wave impedance in a vacuum

### 2.2.1. Definition and limited role

The plane wave impedance in a vacuum is expressed as follows:

$$Z_0 = \left(\frac{E}{H}\right)_{\text{MPPW vacuum}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sim 376,73 \text{ kg m}^2 \text{ s}^{-3} \text{ A}^{-2} \tag{4}$$

Where $\mu_0$ and $\varepsilon_0$ are respectively the dielectric permittivity and the magnetic permeability of free space. The plane wave impedance in a vacuum serves primarily as a reference for other media:

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}.Z_0 \tag{5}$$

Where $\mu_r$ and $\varepsilon_r$ are respectively the relative permittivity and permeability. It is easily verified that the role of plane wave impedance in a vacuum is very limited since it is eliminated in the reflection and transmission coefficients:

$$\mathcal{R} = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{\sqrt{\mu_{r2}/\varepsilon_{r2}} - \sqrt{\mu_{r1}/\varepsilon_{r1}}}{\sqrt{\mu_{r1}/\varepsilon_{r1}} + \sqrt{\mu_{r2}/\varepsilon_{r2}}} \tag{6}$$

Scientific literature generally devotes limited attention to it, and plane wave impedance in a vacuum has so far rarely been the subject of specific analyses. Given the fundamental role of physical constants, which notably form the basis of the definition of units in the 2019 revised International System of Units (SI) [5], this marginal status raises questions and justifies a thorough examination of its physical significance.

### 2.2.2. $Z_0$ is not an electrical impedance

The ratio it represents results in units ($\text{V.m}^{-1}$ divided by $\text{A.m}^{-1}$) that can be converted into ohms, giving it a dimension similar to that of an electrical resistance, which it is not. Plane wave impedance in a vacuum does not represent a voltage/current ratio.

On the other hand, an electrical impedance locally links voltage and current at its terminals, corresponding to a unidirectional, linear configuration, while wave impedance characterizes a fundamentally three-dimensional electromagnetic relationship, and it is only for simplification that we consider only the ratio of the field intensities. More rigorously, the relationship between the electric field and the magnetic excitation field, mutually perpendicular but also perpendicular to the direction of propagation, can be formulated with a tensor electromagnetic impedance, which accounts for this three-dimensional configuration. [4] :

For a unit propagation direction $\vec{n}$:

$$\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \text{ with } {n_x}^2 + {n_y}^2 + {n_z}^2 = 1$$

The vector relationship between the two fields in a vacuum is:

$$\vec{E} = Z_0(\vec{n})\vec{H} \quad \text{with} \quad Z_0(\vec{n}) = \sqrt{\frac{\mu_0}{\varepsilon_0}}.\begin{bmatrix} 0 & n_z & -n_y \\ -n_z & 0 & n_x \\ n_y & -n_x & 0 \end{bmatrix} \tag{7}$$

This represents a fundamental difference between the impedance of a limited, closed electrical circuit and the wave impedance associated with a three-dimensional, unlimited propagation space. The erroneous assimilation to an electrical impedance is only made possible by inappropriate unit conversions and the use, in electromagnetism, of a simplified form of the impedance concept, reduced to the ratio of field amplitudes; this simplification is generally sufficient for common applications.

On the other hand, by definition, no charge circulates in a vacuum and no Joule heating losses occur with the displacement current; there is no absorption, therefore no resistive behavior. There is also no reactive behavior in the case of a progressive plane wave (the setting of the definition of $Z_0$), since, regardless of the frequency, no phase shift appears between the two fields. Therefore, no equivalent electrical scheme can be proposed for wave impedance in a vacuum, which would obviously be possible if $Z_0$ were an electrical impedance.

### 2.2.3. Impedance $Z_0$ is not a property of a vacuum

The impedance $Z_0$ corresponds to a special case of electromagnetic waves, that of plane waves in a vacuum. In the general case, the wave impedance E/H depends on the nature of the electromagnetic wave. Figure 1, shows the components of the fields produced in a vacuum by a short dipole of length L, at a distance r from it [6]:

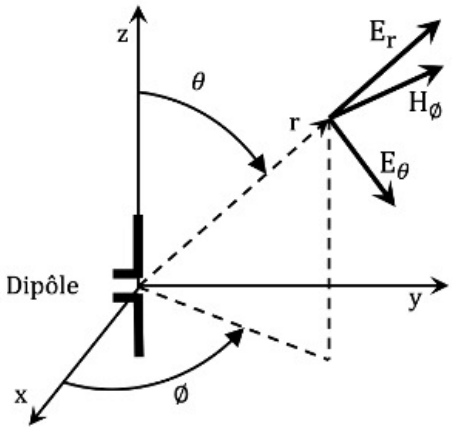



**FIGURE 1**. Components produced by a short dipole

The components of the fields in the orthogonal coordinate system $(r, \theta, \phi)$, are:

$$E_r = \frac{I_0.e^{j\omega(t-r/c)}.L.cos\theta}{2\pi\varepsilon_0}\left(\frac{1}{c.r^2}+\frac{1}{j\omega.r^3}\right)$$

$$E_\theta = \frac{I_0.e^{j\omega(t-r/c)}.L.sin\theta}{4\pi\varepsilon_0}\left(\frac{j\omega}{c^2.r}+\frac{1}{c.r^2}+\frac{1}{j\omega.r^3}\right)$$

$$E_\phi = 0$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = \frac{I_0.e^{j\omega(t-r/c)}.L.sin\theta}{4\pi}\left(\frac{j\omega}{c.r}+\frac{1}{r^2}\right)$$

In the near field, the ratio between the electric field and the magnetic excitation field has an imaginary component, indicating a time phase shift between E and H, and its magnitude varies with the distance from the source, unlike the case of plane waves. The wave impedance E/H in a vacuum is therefore a local ratio that depends on both the wave geometry and the distance from the source. In the far field, this ratio becomes constant and real, and this case is similar to that of plane waves.

$$r \longrightarrow \infty \;\Rightarrow\; \frac{E}{H} \longrightarrow \frac{E_\theta}{H_\phi} \longrightarrow \frac{1}{\varepsilon_0.c} = Z_0 \qquad (8)$$

The impedance of a plane wave in a vacuum does not characterize the vacuum as a material medium, but rather a specific case of waves. It is thus a characteristic of plane waves in the absence of matter, and not a property of the vacuum itself.

The vacuum, as the absence of a material medium, cannot possess intrinsic physical properties. Attributing such properties would amount to reintroducing the hypothesis of a "luminiferous ether," invalidated by the Michelson and Morley experiment (1887).

## 2.3. Relevance of the E/H ratio

Intrinsic impedance is an electric field/magnetic excitation field ratio, but we must now question the relevance of the E/H field ratio rather than D/H, D/B or E/B.

### 2.3.1. For a change of medium

Masao Kitano provides an initial answer to this question regarding a change of medium and the choice of H [7]:

*« The reason why H is used instead of B is as follows. The boundary conditions for magnetic fields at the interface of media 1 and 2 are H1t = H2t (tangential) and B1n = B2n (normal). For the case of normal incidence, which is most important practically, the latter condition becomes trivial and cannot be used. Therefore H is used more conveniently. »*

The same reason can be given for choosing E rather than D, since the boundary conditions are E1t=E2t and D1n = D2n in the absence of free charges. Following this line of reasoning, the E/H ratio is therefore the most appropriate to account for reflection and transmission phenomena during a change of medium.

### 2.3.2. Without changing of medium

When we do not consider the phenomena of reflection or transmission at an interface, but only the effect of the medium on propagation, we find that it is the "phase velocity" parameter that is impacted and that the E/B ratio is appropriate to express it:

$$\left(\frac{E}{B}\right)_{\text{MPPW}} = \frac{1}{\sqrt{\mu.\varepsilon}} = v \qquad (9)$$

Intrinsic impedance is therefore not the only relevant ratio between electric and magnetic fields. Intrinsic impedance is simply the appropriate ratio for studying changes of medium. As for the wave impedance of vacuum, it is only the reference value which is eliminated in the calculation of reflection coefficients.

### 2.4. A role that is difficult to grasp

We saw in 2.2.1 that the plane wave impedance of a vacuum has attracted little interest from physicists and that its interpretation needs to be examined. We saw in sections 2.2.2 and 2.2.3 that $Z_0$ is neither an electrical impedance nor a property of a vacuum. We also saw in 2.3 that $Z_0$ represents a relevant ratio only for changes in medium, but its value is eliminated in the calculations of reflection coefficients. It appears to have no function. We will therefore seek to understand its meaning based on the role of fields.

## 3. NEW APPROACH BASED ON THE ROLE OF THE FIELDS

### 3.1. Physical roles of the D, E, H and B fields

#### 3.1.1. Role descriptions

Electromagnetic fields can have the physical role of a mechanical effect field or that of an excitation field:

- Mechanical effect fields manifest themselves physically as the Lorentz force they exert on a charge. They depend on excitation fields, but also on the media, since polarization or magnetization results from the bound sources within these media. These are the fundamental fields.
- Excitation fields are independent of the medium and correspond to the geometric configuration of free sources. As mathematical tools, they are auxiliary fields.

#### 3.1.2. Physical roles of electric fields

The electric field of mechanical effect is the field E; this can be verified with the electric component of the Lorentz force exerted on a charge $q$:

$$\vec{F}_{elec} = q.\vec{E}$$

The electric excitation field is the field D. A common, incorrect interpretation is to say that it depends on the polarization due to the following relationship:

$$\vec{D} = \varepsilon_0.\vec{E} + \vec{P}$$

Indeed, this relationship is used, for example, in the case of dielectrics separating the plates of a parallel-plate capacitor across which a voltage is applied. In this configuration, the voltage $V$ determines the value of the field $E = V/l$ as a function of the thickness $l$ of the dielectric and consequently determines the polarization, since it is the field E that acts on the charges bound to the material. Thus, this expression gives the illusion that the excitation field D is a consequence of the reaction of the medium, but this is not the case: *« Field D depends only on the loads outside the material, which are controlled by the experimenter. »* [8]. This control of electrical charges is achieved here through voltage:

$$Q = C.V = \frac{\varepsilon.S}{l}.V$$

In the example of the capacitor, the value of the field D depends only on the charges +Q and -Q on each plate and on the area S of the charged surfaces, and not on the medium:

$$D = \frac{Q}{S}$$

Thus, for a given charge value Q, it is the field E and the polarization P that depend on the dielectric separating the plates.

We can also verify, in a simplified way, in the specific case of an isolated charge Q, that at a distance $r$, the expression for the field $E = Q/(4\pi.\varepsilon.r^2)$ depends on the permittivity of the medium, while that of the excitation field $D = \varepsilon.E = Q/(4\pi.r^2)$ corresponds to the geometric configuration of the source independently of the medium. To verify in general that the excitation field D is independent of the medium, we must refer to the Maxwell-Gauss equation in which the permittivity of the medium does not appear:

$$div\ \vec{D} = \rho_f$$

Where $\rho_f$ is the free charge density. The equivalent macroscopic Maxwell-Gauss equation, but formulated with the electric field in an isotropic medium, shows that it is the field E that depends on the medium:

$$div\ \vec{E} = \frac{\rho_f}{\varepsilon}$$

The electric field of mechanical effect E depends on the excitation field and the response of the medium:

$$\vec{E} = \frac{1}{\varepsilon}.\vec{D} = \frac{1}{\varepsilon_0}.(\vec{D} - \vec{P})$$

#### 3.1.3. Physical roles of magnetic fields

The magnetic field with mechanical effect is the B field. This can be verified with the magnetic component of the Lorentz force exerted on a charge $q$ moving at velocity $\vec{v}$:

$$\vec{F}_{mag} = q.\vec{v} \wedge \vec{B}$$

The magnetic excitation field is the field H. This is verified by the macroscopic Maxwell-Ampère equation in the media:

$$\overrightarrow{rot}\ \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

This relationship shows that the excitation field H depends only on free currents and variations in the electric excitation field D, which, as previously shown, itself depends only on free sources. The excitation field H is therefore independent of the medium. The magnetic field of mechanical effect B depends on the excitation field and the response of the medium.

$$\vec{B} = \mu.\vec{H} = \mu_0.(\vec{H} + \vec{M})$$

#### 3.1.4. Summary of the physical roles of the fields

- The E and B fields are the mechanical effect fields. They depend on the excitation fields as well as the media through polarization or magnetization.
- The D and H fields are the excitation fields. They are independent of the media and depend only on free sources.

### 3.2. Electromechanical meaning of the constant $Z_0$

#### 3.2.1. The distinct roles of the fields in the E/H report

The roles of the E field (mechanical effect field) and the H field (excitation field) are not the same, and this ratio was only used for its utility during a change of medium because E1t=E2t and H1t=H2t. Outside of this specific context, a ratio of quantities that are not comparable is not relevant. The meaning of $Z_0$ must be sought in this difference in roles.

Using the E and H fields, the Maxwell-Faraday and Maxwell-Ampère equations in the absence of free currents are as follows in a vacuum:

$$\overrightarrow{rot}\ \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\overrightarrow{rot}\ \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

These equations lead to the expression for the plane wave impedance in a vacuum. In an electromagnetic wave, the electric and magnetic fields do not have distinct sources: they are two components generated simultaneously and coupled via these equations, where the time-varying terms of E and H appear as the effective sources of each other.

Consequently, in the case of a progressive plane wave, these sources cancel each other out in the E/H ratio. We can thus hypothesize that this ratio represents only the mechanical effect specific to the electric field E. Wave impedance in vacuum could then be interpreted as an electromechanical constant, but this must be confirmed by the second field ratio "mechanical effect/excitation", i.e., B/D.

#### 3.2.2. Verification with the B/D ratio

The ratio B/D represents an inversion, for the numerator and denominator, of the electric and magnetic fields, but like E/H, it corresponds to the ratio of an effect field to an excitation field. In a vacuum, the B/D ratio of an electromagnetic wave can be written as:

$$\frac{B}{D} = \frac{\mu_0.H}{\varepsilon_0.E} = {Z_0}^2.\frac{1}{Z_0} = Z_0 \tag{10}$$

We observe that wave impedance in a vacuum is neither a ratio of fields, "electric/magnetic" nor a ratio, "magnetic/electric". It is a ratio of fields, "mechanical effect/excitation". This confirms that it is in fact a constant related to the mechanical effect.

## 4. INTEGRATION OF RECENT DEVELOPMENTS

### 4.1. Recent developments

The inconsistency of the initial laws of electromagnetism is recalled in Appendix A. This inconsistency had been worked around in the SI by the use of constants, but the cause of the inconsistency was not resolved. The constants had been calculated to make these laws consistent, taking into account MKSA units and rationalization. The cause of this initial inconsistency stemmed from the failure to account for the speed of light. This problem was resolved in 2024 [9].

#### 4.1.1. Ampère and Coulomb forces

The resolution in 2024 of the inconsistency in the initial laws of electromagnetism leads to the following expressions for Ampère's electromagnetic force and Coulomb's electrostatic force:

$$\frac{F_A}{L} = K_{MKS-A} \cdot \frac{1}{c} \cdot \frac{I.I\prime}{2\pi.r} \tag{11}$$

$F_A/L$ is the force per unit length exerted on straight, parallel and infinite conductors separated by a distance $r$. $I$ and $I'$ are the currents flowing in the two conductors.

$$F_C = K_{MKS-A} \cdot c \cdot \frac{Q.Q\prime}{4\pi.r^2} \tag{12}$$

Charges $Q$ and $Q$ are separated by a distance $r$.

#### 4.1.2. Constant $K_{MKS-A}$

In both of these expressions, the constant $K_{MKS-A}$ appears. This is the rationalization and adaptation constant to MKSA units of the complete expressions of forces. It is therefore a constant which is the consequence, on the one hand, of the rationalization taking into account the factor $4\pi$ of Heaviside and the adaptation to SI units and on the other hand of the resolution of the initial inconsistency which leads to taking into account the speed of light. Indeed, it is worth recalling that the initial expression for Ampère's force, which served as the basis for defining the units of the CGS-UEM system from which the MKSA units were defined, was as follows:

$$\frac{F_A}{L} = 2.\frac{I.I\prime}{r}$$

The ampere was defined from this expression for a current in the two conductors resulting in a force of $2\ .10^{-7}$ N with $L = r = 1$m. Thus, since they were not taken into account in this initial expression, the factor $4\pi$, the speed of light c, and the coefficient $K_A = 10^{-7}$ N.A$^{-2}$ constitute the constant that rationalizes and adapts the complete expression to SI units:

$$K_{MKS-A} = 4\pi.c.10^{-7}\ \text{N A}^{-2} \tag{13}$$

$$K_{MKS-A} \sim 376,73\ \text{kg m}^2\ \text{s}^{-3}\ \text{A}^{-2}$$

#### 4.1.3. Permeability and permittivity of vacuum

We recognize above the value of the plane wave impedance in a vacuum. Indeed, by identifying these new expressions for Ampère and Coulomb forces with the SI expressions, we obtain:

$$\mu_0 = K_{MKS-A} \cdot \frac{1}{c} \tag{14}$$

$$\frac{1}{\varepsilon_0} = K_{MKS-A} \cdot c \tag{15}$$

Therefore, calculating the plane wave impedance in a vacuum gives:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = K_{MKS-A} \tag{16}$$

Thus the constant $K_{MKS-A}$ is not only the rationalization and adaptation constant to MKSA units of the complete expressions of forces, since it also corresponds to the plane wave impedance in vacuum.

### 4.2. Synthesis

The value of plane wave impedance in a vacuum comes solely from the historical construction of the system of units, but its physical meaning must be synthesized from two observations:

- The constant $Z_0$ is a constant related to the mechanical effects of fields, whether electric or magnetic.
- This constant appears in the expressions for Ampère and Coulomb forces during a mutual effect at a distance of electrical quantities (charges or current).

The plane wave impedance in a vacuum $Z_0$ is related to the mechanical effects of electromagnetic fields. It systematically acts as a multiplier during interactions between two electrical entities (charges or currents). It is therefore an electromechanical coupling constant.

## 5. CONCLUSIONS, EVALUATION, PERSPECTIVES

### 5.1. Conclusions

The plane wave impedance of the vacuum is neither an electrical impedance nor a property of the vacuum; it is as much the ratio E/H as the ratio B/D and is therefore linked solely to the mechanical effects of the E or B fields. These observations, added to its presence in the Coulomb electrostatic force and the Ampère electromagnetic force, show that it is an electromechanical coupling constant.

$$Z_0 = K_{CEM} \tag{17}$$

Not to be confused with the dimensionless electromechanical coupling coefficient, which is involved in the physics of piezoelectric materials.

This constant, rendered invisible in the permeability and permittivity of the vacuum, is of paramount importance because it is involved in all phenomena using these two constants.

### 5.2. Evaluation of the conclusions

A mechanical effect of electromagnetic origin involves either $\mu_0$ or $1/\varepsilon_0$ which are combinations of $Z_0$ and the speed of light c. The systematic involvement of $Z_0$ as a multiplying factor in these phenomena of electromagnetic interactions corroborates its role as a coupling constant between mechanics and electromagnetism.

Dimensional verification of the electromechanical coupling:

The watt and the joule (which corresponds to the watt up to the time value) are the only units common to both electrical and mechanical systems. The ampere is the basic unit of electricity: all electrical units (derived units) are defined from the watt, defined by mechanics, and from the ampere. [5]. In the expression for $Z_0$, the interaction between two electrical entities is expressed with square amperes: $Z_0 \sim 376,73$ kg m$^2$ s$^{-3}$ A$^{-2}$. We also note that the three fundamental mechanical units of this constant correspond precisely to the mechanical expression of the watt (1 kg m$^2$ s$^{-3}$) which is the unit common to the two domains. Thus the dimension of $Z_0$ is:

$$[Z_0] = M.L^2.T^{-3}.I^{-2} = \left[P_{meca}/I^2\right]$$

This confirms its role in coupling between the mechanical and electrical domains for electromagnetic interactions. The expression for $Z_0$ with its units must be:

$$Z_0 \sim 376,73\ \text{W.A}^{-2}$$

The conversion to ohms has historically emptied this constant of its fundamental meaning as much as a conversion from newton-meter to joule would have emptied the moment of a force of its meaning.

### 5.3. Research prospects

The resolution of the inconsistency of the initial laws of electromagnetism opened up research perspectives which concerned on the one hand the role of wave impedance in the static laws of electromagnetism and on the other hand the physical actions that the speed of light represents in the new expressions of these laws. This second question is still asked but the physical meaning of the impedance of the vacuum is determined: it is not specifically an impedance and above all it is neither intrinsic nor characteristic of the vacuum. Its new interpretation as an electromechanical coupling constant is a very important step forward in understanding the new expressions for Ampère and Coulomb forces. Indeed, the physical actions that the speed of light represents in these expressions are, at this stage, merely the result of a mathematical solution without a physical explanation. This work remains to be done, taking this new information into account.

On the other hand, the fact that wave impedance in a vacuum is an electromechanical coupling constant should allow for a better understanding of the electromagnetic relationships involving $\mu_0$ or $1/\varepsilon_0$. For example, the fine-structure constant can be expressed as:

$$\alpha = \frac{e^2}{2.\varepsilon_0.h.c} = \frac{K_{CEM}.e^2}{2h}$$

Yet, the electromechanical coupling constant $K_{CEM}$ has the dimensions of power per square ampere or action per square coulomb (see appendix B). Therefore, the numerator represents a quantity of electromechanical action between two elementary charges, and the denominator represents Planck's constant, which represents the quantum of action.

This example shows that interpreting $Z_0$ as an electromechanical coupling constant $K_{CEM}$ opens up interesting perspectives.

## APPENDICES

## A. Summary of the historical circumvention of the inconsistency of the initial laws

### A.1. The inconsistency between Coulomb's and Ampère's laws

[10]

In 1856, Wilhelm Weber and Rudolph Kohlrausch communicated an observation. The ratio between the measurement of an electric charge based on Coulomb's electrostatic force (UES system) and its measurement based on Ampère's electromagnetic force (UEM system) gave the speed of light:

Electrostatic force (formulation at the time):

$$F_C = \frac{Q.Q\prime}{r^2} \quad \text{Without the factor } K_C = \frac{1}{4\pi\varepsilon_0} \text{ of the SI system} \tag{18}$$

Charges $Q$ and $Q$ are separated by a distance $r$.

Electromagnetic force (formulation at the time):

$$\frac{F_A}{L} = 2.\frac{I.I\prime}{r} \quad \text{Without the factor } K_A = \frac{\mu_0}{4\pi} \text{ of the SI system} \tag{19}$$

$F_A/L$ is the force per unit length exerted on straight, parallel and infinite conductors separated by a distance $r$. $I$ and $I'$ are the currents flowing in the two conductors.

Our electrical units did not yet exist; charges and currents were expressed using millimeters, milligrams, and seconds based on mechanical measurements (measurements described as absolute, as opposed to relative). However, the measurements were inconsistent between the electrostatic system (ESU) and the electromagnetic system (EMU). Indeed, with Coulomb's force, electric charge had the following dimensions:

$$[Q_{UES}] = \left[\sqrt{F_C}.r\right] = M^{\frac{1}{2}}.L^{\frac{3}{2}}.T^{-1}$$

With Ampère's force, the current had the following dimensions:

$$[I_{UEM}] = \left[\sqrt{\frac{F_A.r}{2.L}}.\right] = M^{\frac{1}{2}}.L^{\frac{1}{2}}.T^{-1}$$

Either for the load:

$$[Q_{UEM}] = M^{\frac{1}{2}}.L^{\frac{1}{2}}$$

The ratio of the dimensions of the charges is a speed; the measurement systems were not consistent. But very interestingly, for the same charge, Weber and Kohlrausch obtained a ratio that was equal to the speed of light (measured seven years earlier):

$$\frac{Q_{ESU}}{Q_{EMU}} = c \tag{20}$$

This ratio, along with Faraday's discovery of the magneto-optical effect eleven years earlier, enabled Maxwell to understand the link between electromagnetism and light. He synthesized electromagnetism and demonstrated the propagation of electromagnetic waves; however, the inconsistency between the initial expressions for the forces remained unresolved.

### A.2. A historical circumvention of inconsistency

[11]

From 1874 onwards, units specific to electricity (ampere, volt, ohm, etc.) were created. They were based on Ampère's force. Giovanni Giorgi adapted the two laws to these electrical units and to the MKS units using dimensional constants. Finally, Oliver Heaviside proposed replacing the previous constants with two new constants, $\mu_0$ and $\varepsilon_0$, which he named permittivity and permeability of free space. He proposed this replacement to introduce into the laws a factor $4\pi$ (the solid angle

of all space), which allowed for a simplification of the writing of Maxwell's equations where this factor $4\pi$ disappeared. The problem was thus circumvented through the use of constants, but the way in which the speed of light must be taken into account in the expressions for the forces of Ampère and Coulomb to eliminate this inconsistency was not resolved. This problem was mathematically solved in 2024.

## B. Conventional status of the ampere as the base unit

The concept of base units was rethought in the ninth edition of the SI; indeed, fundamental constants now serve as the basis for units [5]. The ampere was defined, since the advent of the SI, by Ampère's electromagnetic force, with the following definition until the eighth edition [12]:

*« The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to* $2 \cdot 10^{-7}$ *newton per metre of length. »*

The status of the ampere as the basic unit of electricity was factual, since other electrical units, such as the coulomb, were derived from the ampere. The new definition in the ninth edition is as follows:

*« The ampere, symbol* A*, is the electrical current unit of the SI. It is defined by taking the fixed numerical value of the elementary charge, e, equal to* $1,602\ 176\ 634 \cdot 10^{-19}$ *when it is expressed in* C*, a unit equal to* A s*. »*

Thus, we see that it is now the coulomb, through the elementary charge, that has become the actual electrical unit from which the ampere is defined. However, the ampere, through this subtle definition, retains its conventional status as the base unit; this is indeed specified in this ninth edition of the SI: *« The concepts of basic units are retained because they are practical and historically well-established. »* (§ 2.3 Definitions of SI units). Regarding base or derived units, it is also specified: *« This distinction is not necessary in principle because the definitions of all units, whether basic or derived, can be directly established from the seven constants. »*.

Therefore, writing the electromechanical coupling constant using units more suited to the Coulomb force is justified:

$$K_{CEM} \sim 376,73 \text{ kg m}^2 \text{ s}^{-1} \text{ C}^{-2}$$

This results in an action per square coulomb.

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